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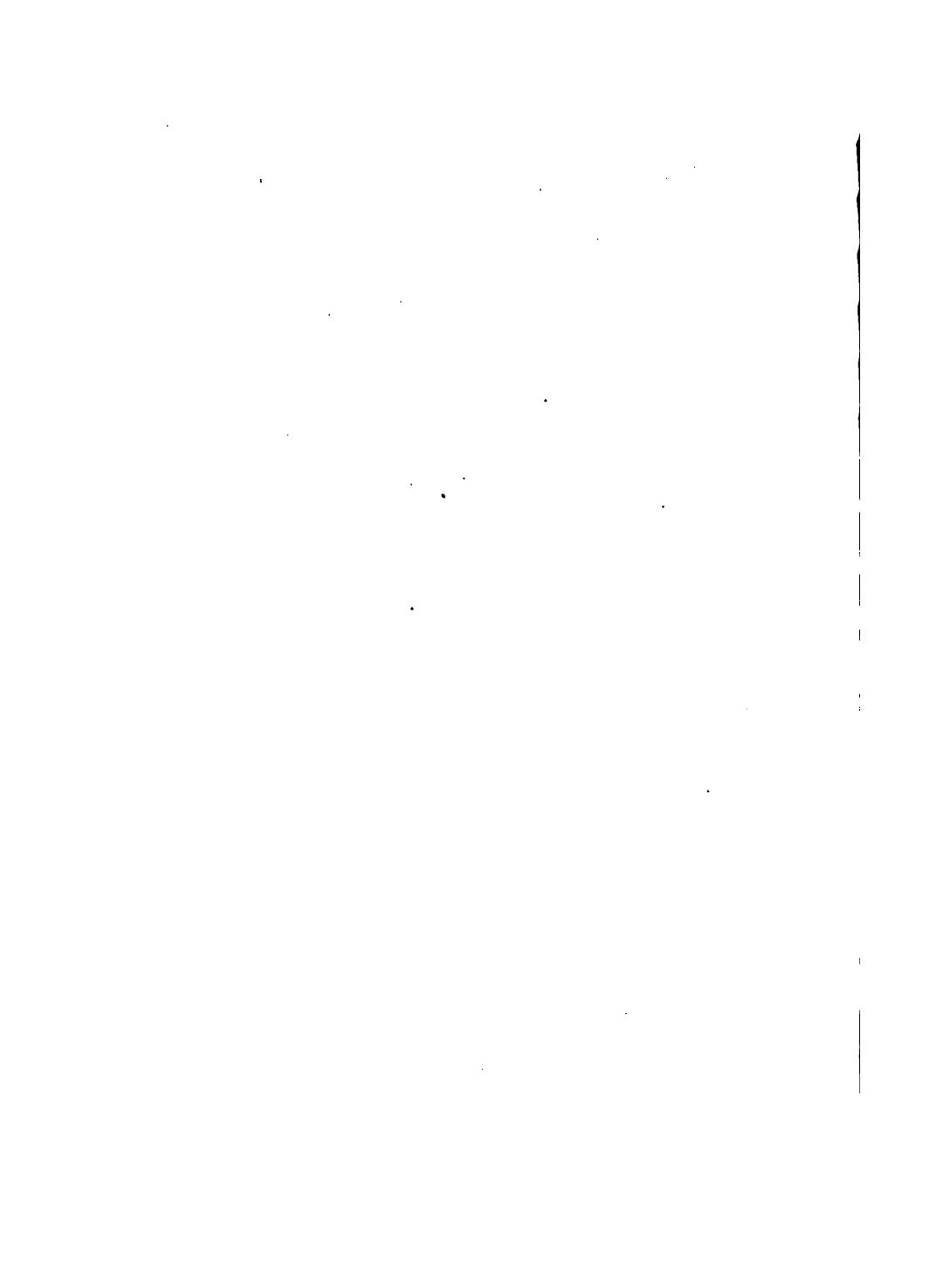
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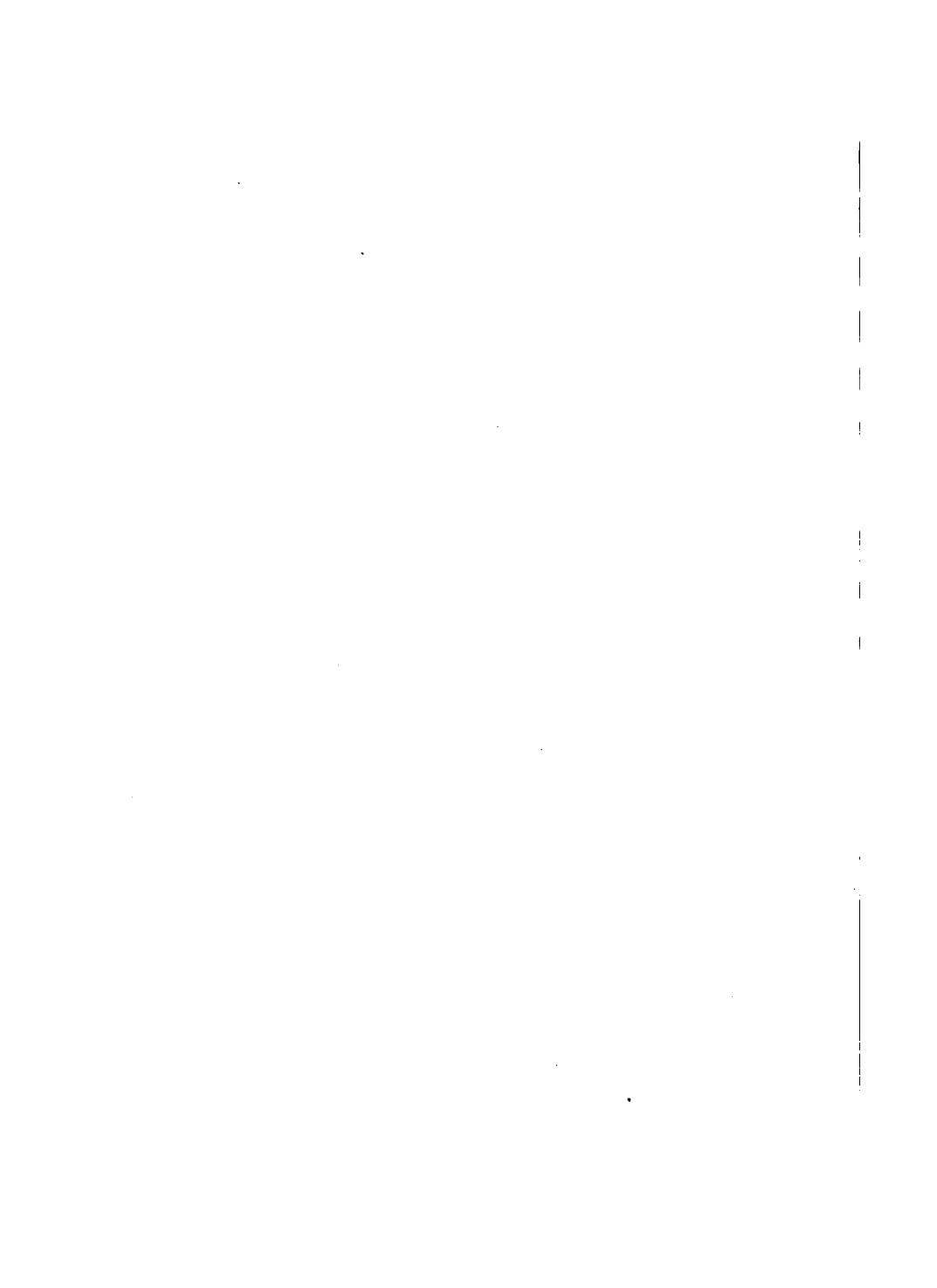
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ELEMENTS OF ARITHMETIC

**FOR PRIMARY AND INTERMEDIATE CLASSES IN
PUBLIC AND PRIVATE SCHOOLS**

BY

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MILNE'S PLANE GEOMETRY—SEPARATE

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E-P 79

PREFACE.

THIS work has been prepared to meet a demand for a book that contains, within brief compass, an elementary course in arithmetic. It is also designed to be an introduction to the more thorough and exhaustive treatment of the science which is given in the author's **STANDARD ARITHMETIC**.

The first part of the book contains exercises arranged in such a manner that they secure an easy and natural development of the ideas of numbers, and they afford sufficient practice to fix the results in memory. Ready knowledge of results can be secured only by frequent repetition and thorough drill, and the lessons that have been prepared are believed to be admirably adapted to produce expertness in computing, without sacrificing interest in the subject through constant practice with abstract numbers.

When the more systematic treatment of the science is presented, the pupil is led by natural, progressive, and logical steps to an understanding of the definitions, principles, processes, and rules, before he is required to state them; consequently, all definitions, principles, and rules are but the expressions of what he already knows. It is evident, therefore, that the plan pursued in the work will develop in the student the habit of investigating for himself any subject which may claim his attention, and this is an extremely important part of proper teaching.

The number of oral examples is large enough to supply all the necessary training in "mental" arithmetic, and the frequent exercises for review will fix in the memory all the knowledge of the science that has been acquired. The treatment of the subjects has been adapted to the comprehension of young pupils, and subjects which are too difficult for them have been omitted; and yet in no instance has accuracy of statement or correctness of process been sacrificed in what has been presented.

Though the treatment of the subjects is brief and elementary, a student who goes no farther in his arithmetical studies than this work will take him, will have a knowledge of the science, sufficient to enable him to perform, with accuracy and intelligence, most of the processes required in ordinary business life.

The author desires to express his indebtedness to George I. Aldrich, A.M., Superintendent of Schools of Newtonville, Mass., for valuable aid in the preparation of the earlier parts of the book, and for important suggestions upon the treatment of subjects throughout the work. His reputation as an originator of rational and scholarly methods of teaching arithmetic is a guarantee of the value of his services.

"The Elements of Arithmetic" is submitted to the public with the hope that it may prove to merit general favor on account of the natural and rational development of the subject, and because of its peculiar adaptation to the class of students for whose use it was written.

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SUGGESTIONS TO TEACHERS.

1. It must be borne in mind that the pupils should first be made familiar with the combinations of numbers, before any explanations or analyses are required.
2. The first part of this book is designed to give pupils the knowledge of numbers; and it is suggested, therefore, that the teacher ask for results only.
3. If explanations of processes or analyses of problems are required, they will divert the attention of the pupils from the results, and thus hinder their progress.
4. The lessons which are given are designed to be typical rather than complete, and the thoughtful teacher will devise additional problems of a character similar to those given in the book.
5. By going over the same lessons frequently the pupil will become familiar with numbers; but there is only one way in which thorough knowledge of numbers can be acquired, and that is by repetition of the combinations within brief intervals of time.
6. The examples should at first be given in connection with objects which can be conveniently obtained. From these the transition is easy to concrete examples, in which the splints, or counters, or buttons, or beans, stand for other

units; but too early drill upon abstract numbers is unfavorable to the development of a proper comprehension of arithmetic, and makes the pupils mechanical rather than thoughtful.

7. It will be found advantageous to encourage the pupils to form examples for themselves. Such a plan gives a pleasing variety to the work of the class, and makes the pupils thoughtful regarding the work. Judicious use of this suggestion will aid in inspiring interest in the work.

8. The work upon the slate, or written work, should come only after the oral work, and it should be given only for purposes of review after the numbers have been thoroughly understood.

9. Care should be taken with the written work, that the children make the figures in accordance with proper models, and that everything is done neatly.

10. Be sure to require the students to answer all the development exercises which precede every subject. These exercises will cause the pupils to apprehend the principles that are employed in the solution of the problems that follow, and they will discourage the tendency on the part of the children to give mechanical or formal solutions.

11. Do not fail with every lesson to review the work of previous days, for it is only in this way that pupils become familiar with results and processes, and success in arithmetical processes is dependent to a great degree upon constant *drill*.

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ELEMENTS OF ARITHMETIC.



1. Hold up your right hand.
Hold up your left hand.
How many hands have you?
Place your right hand behind you.

How many hands can I see now?

Place your left hand behind you.

How many hands are behind you?

How many feet have you?

How many eyes have you?

How many heads have you?

Name one thing you can see on my desk.

Show me two blocks.

Give Annie one block.

Clap your hands once.

Say *ball* twice.

Make a mark on your slate.

Make another mark.

How many marks have you made?

One mark and one mark are how many marks?

Rub out one mark. How many marks are left?

This figure stands for one — 1.

This figure stands for two — 2.

Say one word.

Say two words.

Point to one book.

Point to one school-desk.

Hold up two fingers.

Tell me two boys' names.

Tell me a girl's name.

How many feet has a boy?

How many noses has he?

How many ears has he?

Make as many marks as you have hands.

Make the figure that stands for two things.

Make as many marks as you have feet.

Make the figure that stands for one thing.

If I have two cents and give away one of them, how many shall I have left?

If you should have one apple, and a girl should give you one more, how many apples would you have?

If you did not go to school on Monday and on Friday, how many days were you absent that week?

Make pictures of two squares.

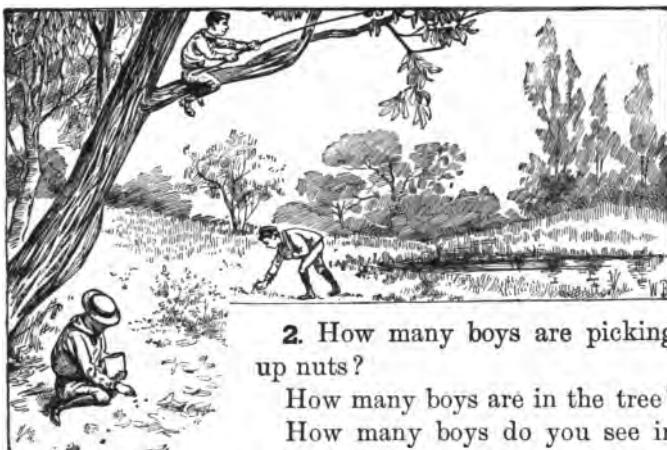
Make pictures of two circles.

Make pictures of two triangles.

and are how many squares?

and are how many circles?

and are how many triangles?



2. How many boys are picking up nuts?

How many boys are in the tree?

How many boys do you see in the picture? If one of the boys should go home, how many would be left?

Show me three pencils.

Make three dots on your slate.

Rub out one dot. How many are left?

Rub out another. How many have you rubbed out?

How many are left?

Now put enough dots with this one dot to make three dots.

One dot and two dots are how many dots?

James gave me two marbles, and Arthur gave me one. How many marbles did they give me?

Two marbles and one marble are how many marbles?

If I lost one of them, how many had I left?

Three marbles less one marble are how many marbles?

I see one bird on the ground, one in a bush, and one flying. How many birds do I see?

This figure stands for three — 3.

Write the figures that stand for one, two, and three.

Tell me what this shows: • • and • are • • •.

□ and □□ are how many squares?

△ and △△ are how many triangles?



3. How many boys do you see in the picture?

How many girls do you see?

How many children are there?

Three boys and one girl are how many children?

Take four blocks. Play that they are little birds.

Four little birds were picking up crumbs. When one flew away, how many were left? Soon afterward, two more flew away. How many had then flown away? How many were left?

How many books has the largest boy in the picture?

How many has the next largest?

If the largest boy should carry all the books, how many would he carry?

Susie and her sister found a nest with four eggs in it. Her sister took two of them. How many were left for Susie?

Four eggs less two eggs are how many eggs?
 In four eggs, how many twos are there?
 How many sides has a square?
 How many sides has a triangle?
 How many triangles can you form with four sticks?
 Try it.

At a cent apiece, how many pears can you buy with four cents?

This figure stands for four—4.
 Write the figures 1, 2, 3, 4 on your slate.
 How many one-cent stamps can you buy for four cents?
 When pencils cost two cents each, what will two pencils cost?

James was two years old, and his sister Grace was two years older. How old was Grace?

Make two squares and write the word *two* beside them. , two.

Make three circles and write the word *three* beside them. , three.

Make four triangles and write the word *four* beside them. , four.

How many squares are and ?
 Here is a shorter way of writing "Two and two are four": $2+2=4$.

The sign + means *and*, and the sign = means *are*.
 Read $2+1=3$; $1+1=2$; $3+1=4$; $1+3=4$.

Copy and complete:

$$1+3=? \quad 2+2=? \quad 1+1=? \quad 1+2=?$$



4. How many fingers have you on one hand?

If you do not count the thumb, how many are there?

Five fingers less one finger are how many fingers?

Point to five children.

Make five *i*'s on your slate.

How many books must I put with four books to make five books?

How many joints has your little finger?

How many joints has your thumb?

How many joints have both your little finger and your thumb?

Three things and two things are how many things?

Mary has five pencils. If only two of them are sharp, how many are dull?

Alice had five yards of ribbon. She gave Sarah three yards of it. How many yards had she left?

Place five splints on your desk in this way: || || |. How many twos are there in five, and how many are left over?

If you had two two-cent pieces and a cent, how much money would you have?

What is another name for five cents?

If you had a half-dime and spent one cent for a pencil and four cents for marbles, how much money would you have left?

This figure stands for five — 5.

Write the figures 1, 2, 3, 4, 5 on your slate.

How many cents are three cents and two cents?

Four cents and one cent? One cent and four cents?
Two cents and three cents?

How much will be left out of five cents if I buy two
two-cent stamps?

It takes five days to go to California. When we have
traveled three days, how many days have we to travel?

I paid three cents for a paper and had two cents left.
How much money had I at first?

Write a figure that means as much as three and two.
Three and one. Two and one. One and one.

How many are



less four apples?

Here is a shorter way of writing "Five less four are
one": $5 - 4 = 1$.

The sign — means *less*. What does = mean?

Copy and complete:

$$5 - 3 = ? \quad 3 - 2 = ? \quad 2 + 3 = ?$$

$$5 - 5 = ? \quad 5 - 4 = ? \quad 4 + 1 = ?$$

$$4 - 1 = ? \quad 5 - 1 = ? \quad 3 + 2 = ?$$

$$4 - 2 = ? \quad 4 - 3 = ? \quad 2 + 2 = ?$$

one 1.

two 2.

three 3.

four 4.

five 5.



5. Cut an apple into two equal parts.

Draw a square and divide it into two equal parts.

How have I divided the circle placed below?

Each of these parts is called a *half*.

How many halves are there in an apple?

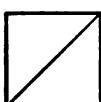
How many halves are there in a square?

Take four blocks. Place them in two equal groups. How many are there in each group? What is the name of each group, or part?

One block is what part of two blocks?

James had two nuts. He gave half of them to me. How many did he give me?

What part of the nuts did he keep?



Divide four oranges equally between Mabel and Annie. What part of the four oranges will each have?

How many of the oranges will each have?

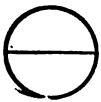
What is half of four books?

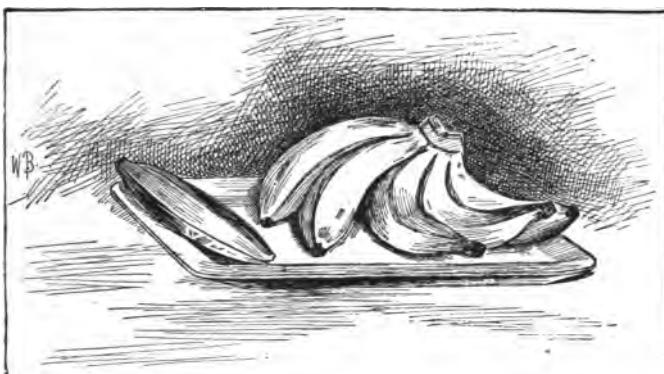
Two cents is half of how many cents?

Mary picked four pints of berries, but spilled half of them. How many did she spill?

Henry made four cents by selling newspapers, and gave half of it to his sister. How much did he give to her?

The wages of a boy were four dollars per week. How much did he earn in half a week? How much in a week and a half?





6. How many bananas do you see in the bunch?
How many bananas are there in the dish?
Five bananas and one banana are how many?
Show me six fingers.
Draw a picture of a pen with six rabbits in it.
Give me one splint; now give me five more. How
many have I now?
Jamie had a half-dime and a cent. To how many
cents was that equal?
Henry had six pears, but he ate one. How many had
he left?
There were six rolls on the plate before supper, and
only one after supper. How many had been eaten?
Johnnie is four years old to-day. How old will he be
in two more years?
Four years and two years are how many years?
It is two miles from Lizzie's house to school, and
four more miles to the station. How far does Lizzie
live from the station?

I must write six letters. When I have written four of them, how many more have I to write?

Make six lines on your slate. Cross out two of them. How many are not crossed out?

How many arms, hands, and feet have you?

How many eyes, ears, and cheeks have you?

Two cents and two cents and two cents are how many cents?

If you had six cents, how many two-cent stamps could you buy with the money?

How much must I pay for three sponges, if each one costs me two cents?

How many boots are there in three pairs of boots?

Henry has six mittens. How many pairs of mittens has he?

Bertie found three eggs in one nest and three in another. How many eggs did he find?

Two threes are how many?

Take six blocks. Give me half of them. How many did you give me?

Nellie had six eggs, but used half of them to make a cake. How many did she use? What part of the eggs was left? How many eggs were left?

In a class of six pupils, half of the children are sitting and half are standing. How many are sitting?

Willie had six cents. He spent it for candy at a cent a stick. How many sticks did he buy?

If he had bought marbles with it at three cents apiece, how many would he have bought?

This is the figure that stands for six — 6.

Write the figures 1, 2, 3, 4, 5, 6, on your slate.

What is the cost of three pencils, at two cents apiece?

SLATE WORK.

$5+1=?$

$6-3=?$

$?+5=6$

$3+2=?$

$2+?=6$

$6-5=?$

$3+?=6$

$4-3=?$

$3+3=?$

$6-?=4$

$6-1=?$

$5-4=?$

$?+2=6$

$6-6=?$

$6-?=2$

$1+3=?$

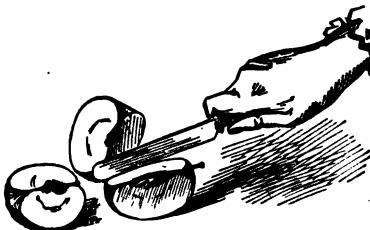
7. How have I divided this circle?
 Draw a line and divide it into three equal parts.



Cut an apple into three equal parts.

Each of these parts is called *a third*.

How many thirds are there in a circle? In a line? In an apple? In anything?



Give me a third of a splint.

Take three pebbles. Divide them into three equal parts. Give me a third of three pebbles.

Now take six pebbles. Divide them into three equal parts. Give me a third of six pebbles.

What is a third of six horses?

If a pie costs six cents, what will a third of a pie cost?

Henry had six apples which he divided equally among three friends. How many did each receive?

School is in session six hours per day, but the little children are not required to attend all the time. If they have one third of the time for play, how many hours do they play?



8. Get a pint measure and a quart measure.

Which is the pint measure? Which is the quart measure?

Fill the pint measure with water. Pour it into the quart measure. Does it fill it?

Fill the pint measure again and empty it into the quart measure. Is the quart measure full now? How many pints of water are there in it?

How many pints of water does it take to make a quart of water? How many pints of milk make a quart of milk? How many pints of berries make a quart of berries?

I have two quarts of chestnuts. How many pint bowls will it take to hold them?

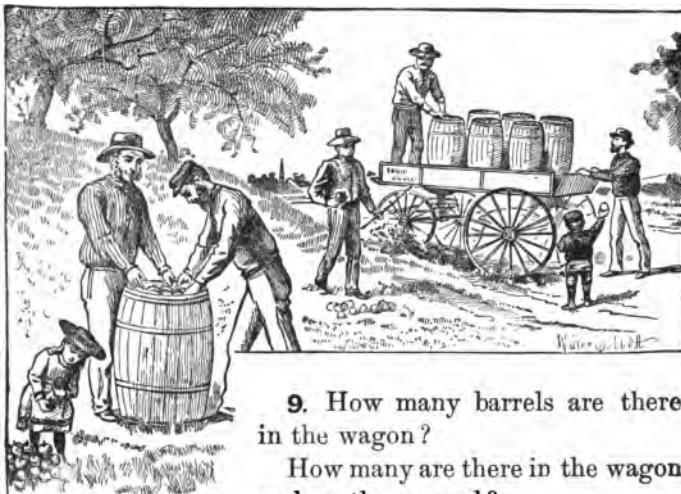
Six pints of oil will fill how many quart cans?

What part of a quart is a pint?

If a quart of milk costs six cents, what will a pint cost?

If a pint of milk costs two cents, what will three pints cost?

Charlie was paid a cent for every pint of cherries he picked. How much did he get for picking three quarts of cherries?



9. How many barrels are there
in the wagon?

How many are there in the wagon
and on the ground?

Six barrels and one barrel are how many barrels?

Name the days of the week. How many are there?

How many would there be if there were no Saturday?

Seven days less one day are how many days?

How many men are busy with the apples?

How many children are trying to help?

Five men and two children are how many people?

If two of the men should go home in the wagon, how many people would be left?

If the two children should then run off to play, how many would be left to work?

How many twos are there in three?

How many twos are there in seven?

There were seven leaves on a twig, and five of them were blown away. How many were left?

Draw a triangle \triangle . Draw a square \square .

How many sides has the triangle?

How many sides has the square?

How many sides have both figures?

How many more sides has the square than the triangle?

A farmer had seven tons of hay. If he sold three tons, how many had he left?

If a pair of boots costs three dollars, how many pairs can you buy with seven dollars? How much will you have left?

A boy bought seven pencils. He fell and broke four of them. How many whole ones had he?

Show with blocks how many fours there are in seven.

How many quart measures can you fill with seven pints? What part of a quart have you left?

What will seven peaches cost, if each one costs a cent?

This is the figure that stands for seven — 7.

If I give two oranges to Lizzie and two to Annie, how many do I give away?

How many times do I give two oranges away?

Two times two oranges are how many oranges?

Two times two is written thus: $2 \times 2 = 4$.

What does the sign \times mean?

Read and complete:

$$3 \times 2 = ? \quad 1 \times 5 = ? \quad 4 \times 1 = ? \quad 3 \times 1 = ?$$

$$7 \times 1 = ? \quad 2 \times 3 = ? \quad 1 \times 7 = ? \quad 2 \times 2 = ?$$

SLATE WORK.

$$6 + 1 = ? \quad 7 - 2 = ? \quad 4 + ? = 7 \quad 3 \times 2 = ?$$

$$1 + 6 = ? \quad 7 - 5 = ? \quad 7 - ? = 3 \quad 2 \times 2 = ?$$

$$7 - 1 = ? \quad 5 + 2 = ? \quad 8 + ? = 7 \quad 2 \times 3 = ?$$

$$7 - 6 = ? \quad 2 + 5 = ? \quad 7 - ? = 4 \quad 7 \times 1 = ?$$



10. This girl has poured seven pints of milk into the dish. If she pours in one pint more, how many pints will then be in the dish?

If she empties the dish by pouring the milk into quart cans, how many cans will it fill?

After she has filled one can, how much is left in the dish?

How many arms have four boys?

How many horns have four cows?

How many chairs must I put with six chairs to make eight chairs?

Fanny had three roses, three lilies, and two pansies. How many flowers had she?

How many threes are found in eight? How much is left?

Edith wrote eight words on her slate, and rubbed out three of them. How many were left?

Five birds and three birds are how many birds?

Three geese and five geese are how many geese?

How many horses can be shod with eight horseshoes?

How many two-cent stamps can be bought with eight cents?

What two equal numbers make eight? • • • •

What four equal numbers make eight? • • • •

A gallon contains four quarts. How many gallons are there in eight quarts?

This figure stands for eight—8.

Place eight blocks in two equal groups. How many are there in each group?

What is half of eight blocks?

Eight sheep were in a field, and half of them got out. How many were left?

Break a splint into halves. Break each half splint into two equal parts. Into how many pieces have you broken the splint?

Each piece is called *a fourth*, or *a quarter*.

How many fourths are there in a pie?

Draw a square on your slate. Divide it into four equal parts. Do it in more than one way.

Divide four pears into four equal groups. What is a fourth of four pears?

Place eight marbles in four equal piles. What part of the marbles is in each pile?

Take a fourth of them in your hand.

A fourth of eight marbles is how many marbles?

A boy who had eight cents spent half of what he had for paper. How much did he spend? How much had he left?

A boy who had eight miles to go traveled one half the distance by bicycle. How far did he travel by bicycle?

A girl who earned eight dollars per month paid one half of it for her necessary expenses. How much did she save?



11. Find a pint measure and a gill measure.

Which is the smaller?

Fill the gill measure with water. Pour it into the pint measure. Do this until you fill the pint measure.

How many gills of water fill the pint measure?

How many gills are there in two pints of vinegar?

How many pints are there in two quarts of water?

How many pints are there in three quarts of water?

In eight pints of water, how many quarts are there?

SLATE WORK.

$2+6=?$

$8-7=?$

$8-5=?$

$8-?=4$

$? \times 2=8$

$8 \times ?=8$

$2 \times 4=?$

$?-1=7$

$?+5=8$

$?+3=8$

$8-?=2$

$4+4=?$

Add:

2	5	1	4	3	4	6
6	3	7	4	2	3	2
—	—	—	—	—	—	—

How much is:

$\frac{1}{2} \text{ of } 4$

$\frac{1}{3} \text{ of } 6$

$\frac{1}{4} \text{ of } 4$

$\frac{1}{2} \text{ of } 2$

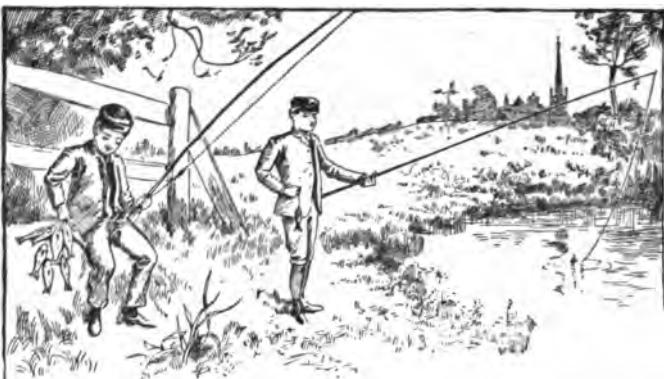
$\frac{1}{3} \text{ of } 3$

$\frac{1}{2} \text{ of } 8$

$\frac{1}{2} \text{ of } 6$

$\frac{1}{4} \text{ of } 8$

$\frac{1}{4} \text{ of } 2 \times 2$



12. David and Fred went fishing.

How many fishes has David, the boy by the fence, on his string? How many has Fred?

How many fishes did both catch?

How many more did David catch than Fred?

How many postal cards can I buy with nine cents, if they cost me a cent apiece?

Emma had a bunch of nine roses, but two of them were lost. How many were left?

If you had a two-cent piece, how much more would you need to make nine cents?

Carl paid three cents for a top, three cents for a pencil, and three cents for a marble. How much did they all cost him?

Three times three cents are how many cents?

Divide nine plums equally among three girls.

What part of the plums will each receive?

What is a third of nine pounds of sugar?

There are six boys and three girls in a class. How many children are there in the class?

How many more boys are there than girls?

If there were nine eggs in a nest and three were taken out, how many were there left?

Ethel had a half-dime and two two-cent pieces. How much money had she?

If she bought oranges at four cents apiece with her money, how many did she buy?

How much shall I have left out of nine dollars, after paying four dollars for a pair of boots?

How many pints are there in nine gills?

How many quarts are there in nine pints?

Frank wrote his name nine times on his slate. He wrote it well only seven times. How many times did he write it carelessly?

How many cents are three cents and six cents more?

If milk costs three cents a pint, how many pints can I buy with nine cents?



How many threes are there in nine?

This figure stands for nine — 9.

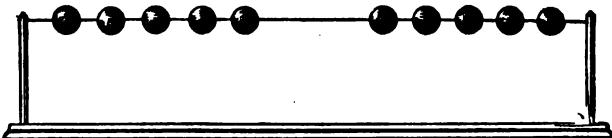
Here are some shorter ways of writing the number of threes in nine:

3's in 9 = 3; or 3) 9 (3; or $9+3=3$.

Read and complete:

2's in 8 = ? 3's in 6 = ? 1's in 7 = ?

4) 8 (?) 6) 6 (?) 1) 5 (?) $9+3=?$ $8+8=?$ $9+1=?$



13. Count the balls on this wire.

What two equal numbers make ten.

Two fives are how many? What is half of ten balls?

Ten cents make a dime. How many cents are there in half a dime?

Jane found a dime. She spent half a dime for a doll. How much had she left?

How many dolls, at that price, could she have bought with the dime?

How many fingers have you?

If you do not count your thumbs, how many have you?

Ten fingers less two fingers are how many fingers?

Place ten blocks in twos. How many twos do you find in ten blocks?

At two cents apiece, what will five bananas cost?

A boy who had ten marbles lost eight of them. How many had he left?

When Carl has read seven pages, how many pages more must he read to make ten pages?

Take four buttons away from ten buttons, and see how many are left?

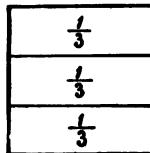
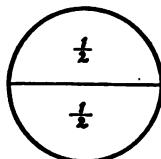
Three threes and how many make ten?

How many squares can you make with ten sticks?

These figures stand for ten — 10.

The figure on the right is called *zero*. What is the left-hand figure called?

Write the figures, 1, 2, 3, 4, 5, 6, 7, 8, 9, 0.



14. What do you call each part of this circle?

One half is written in this way: $\frac{1}{2}$.

What do you call each part of this square?

One third is written in this way: $\frac{1}{3}$.

What do you call each part of this line?



One fourth is written in this way: $\frac{1}{4}$.

Read and complete:

$$\frac{1}{2} \text{ of } 2 = ?$$

$$\frac{1}{2} \text{ of } 10 = ?$$

$$\frac{1}{3} \text{ of } 6 = ?$$

$$\frac{1}{3} \text{ of } 9 = ?$$

$$\frac{1}{3} \text{ of } 4 = ?$$

$$\frac{1}{4} \text{ of } 8 = ?$$

$$3 \text{ is } \frac{1}{2} \text{ of } ?$$

$$2 \text{ is } \frac{1}{3} \text{ of } ?$$

$$2 \text{ is } \frac{1}{4} \text{ of } ?$$

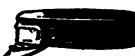
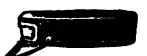
If three bananas cost nine cents, what will each cost?



What part of nine cents are three cents?

What part of nine cents will each banana cost?

If four caps cost eight dollars, how much will each cost?



What part of eight dollars will each cap cost?

15.

REVIEW EXERCISES.

$$8+4=? \quad 6-4=? \quad 2\times 5=? \quad 8+4=?$$

$$6+2=? \quad 8-2=? \quad 3\times 3=? \quad 9+3=?$$

$$5+4=? \quad 10-5=? \quad 4\times 2=? \quad 6+2=?$$

$$2+7=? \quad 5-3=? \quad 1\times 7=? \quad 10+5=?$$

$$4+6=? \quad 9-6=? \quad 5\times 1=? \quad 7+1=?$$

$$1+5=? \quad 7-1=? \quad 3\times 2=? \quad 10+2=?$$

$$\frac{1}{2} \text{ of } 10=? \quad 7-5=? \quad 3+?=5 \quad 5\times ?=10$$

$$\frac{1}{2} \text{ of } 6=? \quad 10-7=? \quad 4+?=9 \quad ?\times 4=8$$

$$\frac{1}{3} \text{ of } 9=? \quad 9-?=5 \quad 6+3=? \quad 9\times 1=?$$

$$\frac{1}{3} \text{ of } 6=? \quad ?-3=5 \quad 10+?=10 \quad \frac{1}{2} \text{ of } 8=?$$

$$\frac{1}{4} \text{ of } 4=? \quad 7+?=10 \quad 8+?=4 \quad \frac{1}{4} \text{ of } ?=2$$

$$\frac{1}{4} \text{ of } 8=? \quad ?+3=9 \quad 9+9=? \quad \frac{1}{3} \text{ of } ?=.1$$

$$2\times 3=? \quad 4+4=? \quad 7-4=? \quad 3+?=9$$

$$8+2=? \quad \frac{1}{2} \text{ of } 2=? \quad 10+10=? \quad 8+?=9$$

$$\frac{1}{2} \text{ of } 4=? \quad 8-4=? \quad 2\times 2=? \quad 5+?=9$$

$$4+2=? \quad 7\times 1=? \quad 6\times 1=? \quad 10-?=2$$

$$9-5=? \quad 3+7=? \quad 6+4=? \quad 10-?=6$$

$$10-6=? \quad 7+2=? \quad \frac{1}{3} \text{ of } 9=? \quad 10-?=1$$

$$\text{Add: } \begin{array}{r} 2 \\ 3 \\ 1 \\ 2 \\ \hline \end{array} \quad \begin{array}{r} 4 \\ 1 \\ 0 \\ 3 \\ \hline \end{array} \quad \begin{array}{r} 1 \\ 2 \\ 3 \\ 4 \\ \hline \end{array} \quad \begin{array}{r} 3 \\ 3 \\ 1 \\ 2 \\ \hline \end{array} \quad \begin{array}{r} 2 \\ 3 \\ 1 \\ 4 \\ \hline \end{array} \quad \begin{array}{r} 4 \\ 1 \\ 2 \\ 1 \\ \hline \end{array}$$

$$\text{From } \begin{array}{r} 9 \\ 3 \\ \hline \end{array} \quad \begin{array}{r} 6 \\ 2 \\ \hline \end{array} \quad \begin{array}{r} 8 \\ 5 \\ \hline \end{array} \quad \begin{array}{r} 7 \\ 3 \\ \hline \end{array} \quad \begin{array}{r} 8 \\ 4 \\ \hline \end{array} \quad \begin{array}{r} 5 \\ 3 \\ \hline \end{array}$$

$$\text{Take } \begin{array}{r} 3 \\ \hline \end{array} \quad \begin{array}{r} 2 \\ \hline \end{array} \quad \begin{array}{r} 5 \\ \hline \end{array} \quad \begin{array}{r} 3 \\ \hline \end{array} \quad \begin{array}{r} 4 \\ \hline \end{array} \quad \begin{array}{r} 3 \\ \hline \end{array}$$

16.



One ten.



One ten and one are eleven; written, /1.



One ten and two are twelve; written, /2.



One ten and three are thirteen; written, /3.



One ten and four are fourteen; written, /4.



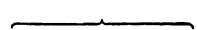
One ten and five are fifteen; written, /5.



One ten and six are sixteen; written, /6.



One ten and seven are seventeen; written, /7.



One ten and eight are eighteen; written, /8.



One ten and nine are nineteen; written, /9.



18. How many eggs are there in this nest?

What other name is given to twelve eggs?

If you should buy a *dozen* oranges, how many would you get?

If they should cost you a cent apiece, how much would you pay for them?

If you should put half of them in a dish, how many would there be in the dish? How many others would there be?

What is another name for six oranges, or six of anything?

There are six working days in a week. How many working days are there in two weeks?

A baby had twelve spools to play with, and one rolled away. How many were left?

If it takes ten yards of lace to trim a dress and two yards to trim an apron, how much will be needed to trim both?

I have six two-cent pieces. How much money have I?

How many two-cent stamps can I buy with a dime and two cents?

James bought a dozen bananas. He ate three, gave three to his brother, and three to his sister. How many had he left?

How many threes are there in twelve?

Place twelve beans in groups of three beans each.

Into how many groups have you separated the twelve beans? What part of the whole is each group?

What is a fourth of twelve miles?

How many months are there in a year? Name them.

If Willie is out of school during June, July, August, and September, how many months in the year does he attend school?

Three horses need how many shoes?

What part of twelve shoes does each horse need?

If the ground is covered with snow during a third of the year, how many months is it covered? How many months in the year is the ground bare?

Five plants in the room are in blossom, and seven are not. How many plants are there in the room?

Take a foot rule. Measure a foot on your desk. Measure the width of a table. See how your rule is divided. What is each of the larger parts of a foot called? How many inches are there in a foot?

If I had a piece of string a foot long and cut off four inches, how many inches would be left?

What number of inches must I add to seven inches to make a foot?

How many inches long is a strap that is just one half of a foot in length?

Six cows have how many horns?

Each of three windows has four panes of glass. How many panes of glass are there in the windows?

SLATE WORK.

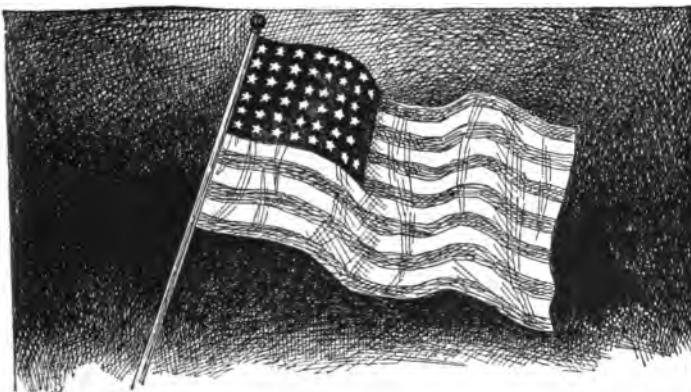
11 + 1 = ?	5 + 3 = ?	12 + 4 = ?	12 + 6 = ?
12 - ? = 11	9 + ? = 12	3 × 4 = ?	? - 7 = 5
? + 11 = 12	8 + 9 = ?	½ of 12 = ?	11 - ? = 8
12 - 11 = ?	? - 9 = 8	7 + ? = 12	? - 6 = 6
? + 2 = 12	4 × 3 = ?	12 - 7 = ?	4 + 6 = ?
2 + 10 = ?	12 + 3 = ?	? - 5 = 7	11 - 7 = ?
12 - ? = 10	½ of 12 = ?	5 + 7 = ?	9 - 6 = ?
? - 10 = 2	8 + ? = 12	6 + 6 = ?	8 × 3 = ?
6 × 2 = ?	12 - 8 = ?	12 - ? = 6	? + 4 = 12
12 + 2 = ?	4 + 8 = ?	½ of 12 = ?	8 + 3 = ?
12 + 5 = ?	12 - ? = 8	2 × ? = 12	10 - 7 = ?
8 + 4 = ?	? × 3 = 12	12 + 7 = ?	? + 9 = 12
12 - ? = 7	5 + ? = 12	12 + 8 = ?	? - 8 = 4
8 + ? = 12	12 - 5 = ?	12 - 4 = ?	? × 6 = 12
12 - 3 = ?	4 × ? = 12	? + 5 = 12	8 + ? = 12
12 + ? = 2	7 + 5 = ?	? + 6 = 12	? - 7 = 5

Add:

4	1	3	1	2	3	6
3	5	4	2	3	4	1
2	4	4	8	4	3	4
—	—	—	—	—	—	—

From	11	9	10	11	10	11
Take	8	5	7	5	2	7

Six 2's = ?	Two 6's = ?	Two 3's = ?	2) 6 (?)
Three 4's = ?	Four 3's = ?	Three 2's = ?	4) 12 (?)
Five 2's = ?	Three 3's = ?	Four 2's = ?	?) 12 (2)



19. Count the number of stripes in the flag.

What colors are the stripes? How many red ones are there? How many white ones?

If one of the red ones should be torn off, how many stripes would be left?

Byron placed eleven blocks in one group and two in another. How many blocks were there in both groups?

If he should put one of the eleven blocks with the other group, how many would there be in both groups? How many in each group?

What is the largest piece of money that can be used in paying thirteen cents?

How much more is needed to make thirteen cents? I had thirteen cents, and I spent four cents for stamps. How many cents had I left?

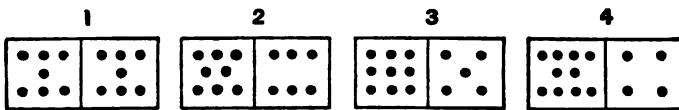
Four two-cent stamps and a five-cent stamp cost how much?

I walked six miles yesterday and seven miles to-day. How many miles did I walk in all?

SLATE WORK.			Supply the Signs.		
$7+6=?$	$10+3=?$	$13-?=6$	8	4	12
$13-6=?$	$13-3=?$	$18-?=2$	7	2	9
$13-7=?$	$13-10=?$	$8+?=18$	11	5	6
$6+7=?$	$3+10=?$	$3+?=18$	8	2	4
$8+5=?$	$11+2=?$	$18-?=4$	4	2	8
$13-8=?$	$13-2=?$	$13-?=10$	4	2	6
$13-5=?$	$13-11=?$	$18-?=8$? + ? = 18		
$5+8=?$	$2+11=?$	$9+?=18$? - 5 = 5		
$9+4=?$	$13 \times 1=?$	$2+?=18$? - ? = 9		
$13-4=?$	$1 \times 13=?$	$5+?=18$? \times ? = 9		
$13-9=?$	$13+1=?$	$18-?=12$? + ? = 2		
$4+9=?$	$13+13=?$	$18-?=7$? + ? = 12		

Add:

2	8	8	5	4	6	2
5	4	2	6	2	4	5
6	5	8	1	6	8	6
-	-	-	-	-	-	-
5	6	7	5	6	5	4
5	8	4	5	5	8	4
2	2	2	1	2	0	4
-	-	-	-	-	-	-
From	18	18	18	12	18	12
Take	8	2	6	10	9	7
	—	—	—	—	—	—
From	12	13	13	18	12	18
Take	5	7	4	5	9	8
	—	—	—	—	—	—



20. How many dots are there in each of these figures?

What two numbers make fourteen in the first figure?
In the second? In the third? In the fourth?

How many days are there in two weeks?

How many more days are there in two weeks than in
one week?

At seven cents apiece, how many tops can I buy with
fourteen cents?

Fourteen geese went down to the pond. Seven of
them soon came back. How many remained at the pond?

With your blocks, count by twos to fourteen.

Begin with one block, and count by twos to thirteen.

I have fourteen cents in two-cent pieces. How many
two-cent pieces have I?

If you had a dime, how many more cents would it
take to make fourteen cents?

How many cents do you need to put with a half-dime
to make fourteen cents?

How many pints are there in seven quarts?

How many quarts are there in fourteen pints?

A foot and two inches are how many inches?

Robert sharpened fourteen pencils for me. The
points of two of them were broken. How many were
left with good points?

I have two three-cent pieces. How many more cents
will it take to make fourteen cents?

Marie is 8 years old to-day. In how many years will
she be 14?

SLATE WORK.

Supply the Signs.

$2 \times 7 = ?$	$7 + ? = 14$	$5 \ 9 \ 14$	$? + ? = 14$
$6 + 8 = ?$	$? - 10 = 4$	$14 \ 8 \ 6$	$? - ? = 10$
$14 - 9 = ?$	$? + 5 = 14$	$12 \ 2 \ 14$	$? \times ? = 14$
$\frac{1}{2}$ of $14 = ?$	$14 - ? = 11$	$14 \ 4 \ 10$	$? + ? = 7$
$7)14(?$	$14 - ? = 2$	$14 \ 7 \ 7$	$? + ? = 12$
$7 \times 2 = ?$	$5 \times 2 = ?$	$12 + 6 = ?$	$11 - ? = ?$
$2)14(?$	$? \times 2 = 12$	$12 + 4 = ?$	$12 - ? = ?$
$1)14(?$	$4 \times 2 = ?$	$12 + 3 = ?$	$13 - ? = ?$
$14)14(?$	$8 \times ? = 6$	$12 + 2 = ?$	$14 - ? = ?$
$14 \times 1 = ?$	$3 \times 3 = ?$	$12 + 1 = ?$	$10 - ? = ?$

Add:

2	5	3	4	1	6	2	3
3	4	7	3	3	2	7	4
7	5	4	7	8	6	5	7
—	—	—	—	—	—	—	—
From	14	13	14	12	14	13	
Take	8	5	10	7	9	8	
	—	—	—	—	—	—	

21. Take your foot rule and draw a line on the board just as long as the rule. How long is the line?

Add to this line another of the same length, and then another. How long is the whole line now?

Take the yard-stick and measure the line with it. How does it compare in length with a yard?

How many feet are there in a yard of anything?

Measure off a yard on the edge of the table.

What is always measured by the yard?

How many feet are there in two yards of string?

A room twelve feet long will need how many yards of carpet to extend the length of the room?

22. How many rows of squares are there in this figure?

How many squares are there in each row?

How many squares are there in the three rows?

Tell from the picture how many threes make fifteen? How many fives?

I have a dime and a half-dime. For how many cents can I exchange them?

Look at the figure given above and tell me what a third of fifteen squares is.

In fifteen feet, how many yards are there?

There were fifteen bananas in a bunch. After ten were sold how many were left?

What three pieces of money make fifteen cents?

If a pound of beef costs ten cents, what will a pound and a half cost?

Count by threes to fifteen.

Write on your slate in a vertical line enough threes to make fifteen. How many did you write?

Five triangles have how many sides?

A dozen and three oranges are how many oranges?

Johnny is fifteen months old. How many months more than a year is that?

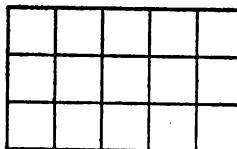
I bought three oranges for fifteen cents. How much did I pay for each orange?

Three times what number equals fifteen?

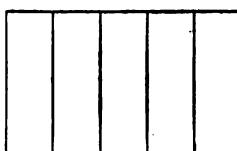
Five times what number equals fifteen?

A boy ran on three errands, receiving five cents for each. How much did he earn?

How many eggs are a dozen and a quarter eggs?



23. Make a figure like the one on the previous page without the two inside horizontal lines.



Into how many equal parts is this figure divided?

Each of these parts is called a *fifth*.

How many fifths are there in the figure which you drew?

How many fifths are there in any figure? In any line?

If a pie were divided into five equal parts and you were given one of the parts, what part of the pie would you receive?

Now you may draw the two inside horizontal lines as shown in the previous figure. How many squares have you now made?

How many squares are there in a fifth of the figure?

What is a fifth of fifteen sheep?

Separate ten blocks into five equal parts. What is a fifth of ten blocks?

This is the way we write a fifth: $\frac{1}{5}$.

SLATE WORK.

$$10 + 5 = ? \quad 15 + 3 = ? \quad 7 + ? = 15 \quad 15 + ? = 5$$

$$15 - 5 = ? \quad 5 \times 3 = ? \quad 9 + ? = 15 \quad 15 + ? = 3$$

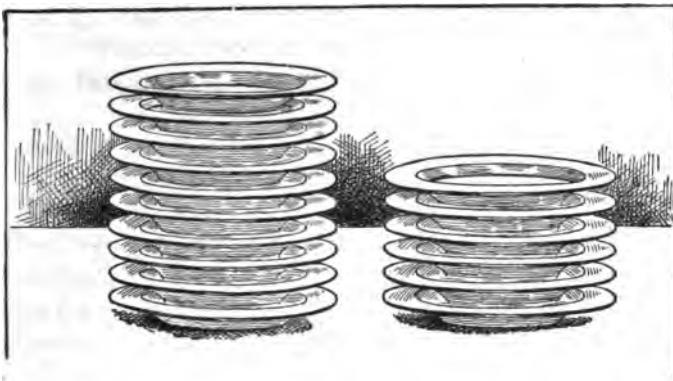
$$15 - 10 = ? \quad \frac{1}{5} \text{ of } 15 = ? \quad 15 - ? = 11 \quad 15 + ? = 1$$

$$5 + 10 = ? \quad 15 - 3 = ? \quad 15 - ? = 5 \quad 15 + ? = 15$$

$$3 \times 5 = ? \quad 15 - 12 = ? \quad 13 + ? = 15 \quad 3 \times ? = 15$$

$$\frac{1}{3} \text{ of } 15 = ? \quad 12 + 3 = ? \quad 15 - ? = 8 \quad 5 \times ? = 15$$

$$15 + 5 = ? \quad 8 + 12 = ? \quad 8 + ? = 15 \quad 1 \times ? = 15$$



24. Count the plates in these two piles.
How many are there in each pile?
How many are there in both piles?
If these plates were in two equal piles, how many would there be in each?

Take sixteen pieces of paper and imagine that they are plates. Place them in two equal piles. What is one half of sixteen plates?

There were sixteen children in a class. One stormy day one half of them were absent. How many were present?

How many shoes do four horses need?
How many oxen can be shod with sixteen shoes, if each ox requires eight shoes?

With your blocks count by twos to sixteen.
Write on your slates in a vertical line a sufficient number of twos to make sixteen. How many twos are there in sixteen?

Begin with sixteen and count backward by twos.
There are four quarts in a gallon. Prove it.

- How many quarts are there in two gallons?
- How many quarts are there in three gallons?
- In sixteen quarts, how many gallons are there?
- What part of a gallon is a quart?
- How many pints are there in a gallon?

SLATE WORK.

$4 \times 4 = ?$	$\frac{1}{2}$ of 16 = ?	$16 - 9 = ?$	$8 + ? = 16$
$8 \times 2 = ?$	$\frac{1}{4}$ of 16 = ?	$16 - 4 = ?$	$? - 8 = 8$
$2 \times ? = 16$	$8 + ? = 16$	$16 - 10 = ?$	$? + 5 = 16$
$16 + 2 = ?$	$16 - 7 = ?$	$16 - 12 = ?$	$? + 9 = 16$
$16 + 4 = ?$	$? + 4 = 16$	$16 - 5 = ?$	$16 - ? = 5$
$16 + 8 = ?$	$10 + 6 = ?$	$16 - 6 = ?$	$16 - ? = 3$

Addi

3	4	8	5	6	4
4	5	2	6	3	2
2	3	2	3	2	3
7	3	4	2	4	4

Subtract:

$$\begin{array}{cccccc}
 16 & 16 & 15 & 15 & 16 & 16 \\
 \hline
 10 & 15 & 8 & 4 & 9 & 5
 \end{array}$$

Answer instantly :

7 + 8	15 + 5	$\frac{1}{2}$ of 16	3 x 3
9 + 4	16 + 1	$\frac{1}{2}$ of 8	4 x 4
8 - 5	16 + 8	$\frac{1}{2}$ of 4	2 x 2
16 - 9	16 + 4	$\frac{1}{2}$ of 2	3 + 8
14 - 11	16 + 2	$\frac{1}{2}$ of 14	8 + 8
12 + 4	14 + 2	$\frac{1}{2}$ of 6	5 + 8
3 x 4	10 + 2	$\frac{1}{2}$ of 12	11 - 7
8 x 2	12 + 2	16 - 12	13 - 6



25. Into how many parts is this line divided? How do these parts compare in size?

Each of these equal parts is called *an eighth*.

How many eighths are there in this square?

Point to an eighth of this square.

An eighth is written in this way — $\frac{1}{8}$.

Separate eight blocks into eight equal parts.



What is an eighth of eight blocks?

If you should divide sixteen pears equally among eight boys, what part of them would each boy get?

How many would each boy get?

What is an eighth of sixteen horses?

A farmer had sixteen tons of hay. If he should sell an eighth of it, how many tons would he have left?

Draw a square. Divide it into halves. Now divide your square into four equal parts.

What is each part called?

How many fourths are there in the square?

How many fourths are there in half of the square?

What, then, is another name for half of the square?

What other name might we give to two fourths of a pie?

Now divide this square into eight equal parts. What is each part called?

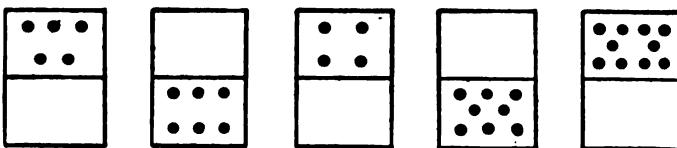
How many eighths are there in the square?

How many eighths are there in half of the square?

How many eighths are there in a fourth of the square?

Tell me all the names you know for a half of anything.

Give me one other name for a fourth of anything.



26. Copy these figures on your slate.

Fill in the blank half of each with enough dots to make seventeen in each one.

Name three pieces of money that will make seventeen cents.

There were seventeen pine trees in a field. If seven of them were chopped down, how many were left standing?

In front of my house there are nine maple trees on one side of the street and eight on the other. How many are there on both sides?

It is seventeen miles from one station to the next. When the train has gone eleven miles from one station, how many miles has it to go before it reaches the other?

A house has six windows in one side, seven in another, and two in each end. How many windows are there in the house?

A man earned seventeen dollars a week and spent twelve dollars of it. How much had he left?

Mr. White sold his turkeys for seventeen dollars. How many sheep at five dollars apiece could he buy with the money? How much would he have left?

A newsboy who bought seventeen papers sold all but five of them. How many did he sell?

Two boys start from the same place. One walks eight miles in one direction, the other walks nine miles in the other direction. How far are they apart then?

SLATE WORK.

17 - 2 = ?	12 + ? = 17	17 + 5 = ?	15 = 17 - ?
17 - 8 = ?	17 - ? = 7	17 + 10 = ?	17 = 10 + ?
17 + 2 = ?	17 - ? = 18	17 + 8 = ?	17 = 2 × 8 + ?
17 + 3 = ?	? + 8 = 17	17 + 6 = ?	17 = 3 × 5 + ?
17 + 4 = ?	6 + ? = 17	17 + 9 = ?	17 = 7 × 2 + ?
7 + 10 = ?	17 - ? = 10	17 + 7 = ?	17 = 4 × 3 + ?

Add:

2	5	4	8	6	9	4
3	4	8	7	2	1	3
6	8	2	5	3	2	8
4	5	2	2	5	3	2
—	—	—	—	—	—	—

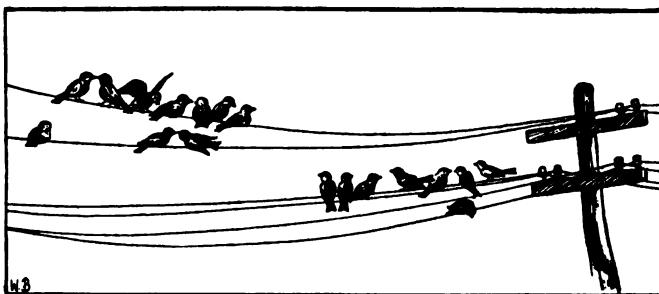
Add:

12	15	8	7	6	5	4
5	2	9	10	11	12	13
—	—	—	—	—	—	—
9	8	9	6	7	5	8
6	5	4	8	6	5	8
—	—	—	—	—	—	—

Out of	17	14	15	16	17	17
Take	8	8	10	7	16	3
	—	—	—	—	—	—

Answer instantly:

9 + 4	12 - 5	8 × 5	1) 16 (
7 + 8	15 - 8	7 × 2	2) 16 (
6 + 11	17 - 9	4 × 3	4) 16 (
4 + 8	16 - 12	8 × 3	8) 16 (
5 + 9	11 - 5	6 × 2	16) 16 (
9 + 8	18 - 8	4 × 4	5) 15 (
6 + 4	14 - 7	2 × 5	7) 14 (



27. How many birds are resting on these wires?

How many are there in each group?

If one from the larger group should join the smaller group, how many would there be in each group then?

There are eighteen windows in a house. If ten of them are in the sides of the house, how many are there in the ends?

Mary's rose-bush had eighteen rose-buds on it, and she picked twelve of them. How many rose-buds were left on the bush?

Lizzie made three button-holes each day for a week, beginning on Monday. How many had she made by Saturday night?

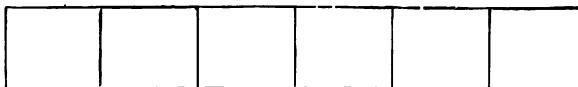
At three cents apiece, how many peaches can I buy with eighteen cents?

Fred learned six verses of a poem each day for three days. How many verses did he learn?

What part of the eighteen verses did he learn each day?

A dozen and a half eggs are how many eggs?

If lace is worth a cent an inch, what will eighteen inches of it cost?



28. Into how many parts is this figure divided?

Each part is called *a sixth*.

How many sixths are there in it?

How many sixths are there in a pie? In a melon?

A sixth is written in this way — $\frac{1}{6}$.

Make a picture of a pie and divide it into sixths. How many sixths are there in half a pie? In one third?

If you cut a card that is six inches long into sixths, how long will each piece be?

What is a sixth of twelve words?

John's overcoat cost eighteen dollars. His hat cost one sixth as much. What did his hat cost?

Draw a line. Divide it into halves, and then into sixths.

Draw another line of the same length. Divide it into thirds, and then into sixths.

How many sixths are there in half of the line?

How many sixths are there in a third of the line?

One half is how many fourths? How many sixths?

How many eighths?

Express with figures one sixth, two sixths, five sixths, three sixths.

Write all the expressions that you know which mean the same as one half.

Write all the expressions that you know which mean the same as one fourth.

How many sixths are 1 sixth and 3 sixths?

How many eighths are 3 eighths and 4 eighths?

SLATE WORK.

$18+2=?$	$6 \times ?=18$	$18-6=?$	$\frac{1}{8} \text{ of } 16=?$
$18+3=?$	$3 \times ?=18$	$18-9=?$	$\frac{1}{4} \text{ of } 16=?$
$18+6=?$	$9 \times ?=18$	$18-12=?$	$\frac{1}{2} \text{ of } 16=?$
$18+9=?$	$2 \times ?=18$	$18-15=?$	$\frac{1}{3} \text{ of } 15=?$
$\frac{1}{2} \text{ of } 18=?$	$9+?=18$	$18-7=?$	$\frac{1}{3} \text{ of } 12=?$
$\frac{1}{3} \text{ of } 18=?$	$10+?=18$	$18-3=?$	$\frac{1}{3} \text{ of } 6=?$
$\frac{1}{6} \text{ of } 18=?$	$8+?=18$	$18-5=?$	$\frac{1}{3} \text{ of } 9=?$
$12+?=18$	$9+8=?$	$\frac{1}{3} \text{ of } ?=2$	$11-7=?$
$18-?=2$	$14-6=?$	$\frac{1}{3} \text{ of } ?=6$	$8+4=?$
$6+?=18$	$6+9=?$	$\frac{1}{3} \text{ of } ?=4$	$14+7=?$
$18-?=14$	$8+5=?$	$\frac{1}{3} \text{ of } ?=3$	$6 \times 3=?$
$18-?=10$	$5+6=?$	$\frac{1}{3} \text{ of } ?=5$	$4+9=?$
$7+?=18$	$9+7=?$	$\frac{1}{3} \text{ of } ?=1$	$\frac{1}{3} \text{ of } 15=?$

Add:

4	5	2	8	6	7	6
3	2	9	7	4	1	3
6	7	2	2	5	4	4
5	4	4	4	3	5	4
-	-	-	-	-	-	-
8	4	5	4	5	5	6
5	6	8	1	1	2	2
7	2	2	3	8	7	5
2	8	1	6	4	2	1
-	-	-	-	-	-	-

Subtract:

18	18	18	18	18	18
17	10	14	9	7	5



29. How many doves do you see in this picture?

What two numbers make nineteen?

Take nineteen blocks. I will name one part of nineteen, and you may find out the other part with your blocks: 12; 6; 8; 15; 5; 16; 7; 9; 11; 4; 13; 10; 14; 3; 17.

Two weeks and five days are how many days?

Alice is nineteen years old. How old was she ten years ago?

There were nineteen cars at the station. After an engine had drawn away twelve of them, how many were left?

Out of nineteen pupils, five were absent. How many were present?

In a house there are nineteen rooms. If eight are upstairs, how many are downstairs?

How many pairs of boots at two dollars a pair can I buy with nineteen dollars? How much shall I have left?

How many triangles can you make with nineteen sticks? How many sticks will be left?

How many are nineteen less ten?

How many gallon cans can be filled with nineteen quarts of milk? How many quarts will be left?

How many five-cent stamps can you buy with nineteen cents? How many cents will be left?

SLATE WORK.

$10+9=?$	$19+9=?$	$13+?=19$	$19=3 \times 5+?$
$12+7=?$	$19+4=?$	$19-?=10$	$19=4 \times 4+?$
$19-8=?$	$19+7=?$	$11+?=19$	$19=5 \times 2+?$
$19-6=?$	$19+5=?$	$19-?=14$	$19=3 \times 6+?$
$14+5=?$	$19+8=?$	$8+?=19$	$19=7 \times 2+?$
$19-7=?$	$19+6=?$	$19-?=9$	$19=2 \times 9+?$

Answer instantly:

8×6	$8+9$	2×8	$\frac{1}{2}$ of 12
$15+5$	$5+9$	2×5	$\frac{1}{2}$ of 8
$13-6$	$6+9$	2×3	$\frac{1}{2}$ of 18
$\frac{1}{6}$ of 18	$4+9$	2×9	$\frac{1}{2}$ of 16
$7+8$	$11+9$	2×7	$\frac{1}{2}$ of 10
5×3	$15-9$	2×4	$\frac{1}{4}$ of 16
$14+7$	$19-9$	2×6	$\frac{1}{4}$ of 8
$17-9$	$14-0$	3×5	$\frac{1}{4}$ of 12
$\frac{1}{2}$ of 12	$18-9$	3×2	$\frac{1}{8}$ of 8
$8+6$	$12-9$	3×4	$\frac{1}{8}$ of 16

Add:

2	6	8	4	3
3	4	5	4	3
5	2	6	4	3
4	3	4	4	3
5	4	1	3	6

30.

REVIEW EXERCISES.

$1+5=?$	$11+5=?$	$9-4=?$	$19-4=?$
$8+6=?$	$13+6=?$	$9-8=?$	$19-8=?$
$5+4=?$	$15+4=?$	$7-5=?$	$17-5=?$
$2+6=?$	$12+6=?$	$7-3=?$	$17-3=?$
$4+4=?$	$14+4=?$	$5-4=?$	$15-4=?$
$6+2=?$	$16+2=?$	$5-2=?$	$15-2=?$
$8+4=?$	$13+4=?$	$8-7=?$	$18-7=?$
$7+2=?$	$17+2=?$	$8-5=?$	$18-5=?$
$4+2=?$	$14+2=?$	$6-4=?$	$16-4=?$
$5+3=?$	$15+3=?$	$6-1=?$	$16-1=?$
$2+7=?$	$12+7=?$	$4-3=?$	$14-3=?$
$8+5=?$	$13+5=?$	$4-2=?$	$14-2=?$
$4+3=?$	$14+3=?$	$8-4=?$	$18-4=?$
$5+1=?$	$15+1=?$	$8-6=?$	$18-6=?$
$1+8=?$	$11+8=?$	$7-6=?$	$17-6=?$
$2+4=?$	$12+4=?$	$7-4=?$	$17-4=?$
$8+2=?$	$13+2=?$	$7-7=?$	$17-7=?$
$4+1=?$	$14+1=?$	$9-7=?$	$19-7=?$
$1+3=?$	$11+3=?$	$9-3=?$	$19-3=?$
$4+5=?$	$14+5=?$	$6-5=?$	$16-5=?$
$6+3=?$	$16+3=?$	$6-2=?$	$16-2=?$
$1+7=?$	$11+7=?$	$3-2=?$	$13-2=?$
$1+4=?$	$11+4=?$	$4-1=?$	$14-1=?$
$8+3=?$	$11+6=?$	$9-6=?$	$13-5=?$
$6+1=?$	$13+3=?$	$7-2=?$	$14-4=?$
$8+5=?$	$12+5=?$	$8-3=?$	$15-6=?$
$7+7=?$	$15+2=?$	$9-5=?$	$18-9=?$



31. Take ten splints and tie them together. Take ten more, and make another bundle. How many tens have you?

Two tens are called *twenty*. They are written *20*.

Take another bundle of ten. How many tens have you now?

Three tens, called *thirty*, are written *30*; four tens, *forty*, are written *40*; five tens, *fifty*, are written *50*; six tens, *sixty*, are written *60*; seven tens, *seventy*, are written *70*; eight tens, *eighty*, are written *80*; nine tens, *ninety*, are written *90*.

Count by tens to ninety.

Write the numbers from ten to ninety on your slate.

Begin at 90 and count back by tens to 0.

Put away all but two bundles of tens. Ten and ten are how many?

How many fives are there in 10? How many in 20?

How many twos are there in 10? How many in 20?

Count by 2's to 20, writing the numbers on your slate, thus: *0, 2, 4*, etc.

Count by 4's in the same way. Count by 5's in the same way.

A boy had 20 quarts of berries and sold one half of them. How many did he sell? How many had he left?

There were 15 birds on a tree. How many more must come to make 20?

How many dimes can you get for 20 cents?

How many half dimes can you get for 20 cents?

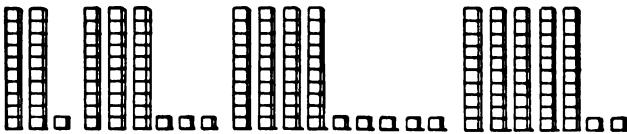
There are two pints in a quart. How many quarts are there in 20 pints?

Four quarts make a gallon. How many gallons are there in 20 quarts?

SLATE WORK.

$10 - 7 = ?$	$10 - 6 = ?$	$\frac{1}{2}$ of 10 = ?	$1 \times 2 = ?$
$20 - 7 = ?$	$20 - 6 = ?$	$\frac{1}{2}$ of 20 = ?	$2 \times 2 = ?$
$10 - 4 = ?$	$10 - 5 = ?$	$\frac{1}{2}$ of 10 = ?	$3 \times 2 = ?$
$20 - 4 = ?$	$20 - 5 = ?$	$\frac{1}{2}$ of 20 = ?	$4 \times 2 = ?$
$10 - 2 = ?$	$20 + 5 = ?$	$10 - 1 = ?$	$5 \times 2 = ?$
$20 - 2 = ?$	$20 + 2 = ?$	$20 - 1 = ?$	$6 \times 2 = ?$
$10 - 8 = ?$	$1 \times 5 = ?$	$20 + 4 = ?$	$7 \times 2 = ?$
$20 - 8 = ?$	$2 \ 5's = ?$	$20 + 10 = ?$	$8 \times 2 = ?$
$10 - 3 = ?$	$3 \ 5's = ?$	$10 + 4 = ?$	$9 \times 2 = ?$
$20 - 3 = ?$	$4 \ 5's = ?$	$10 + 10 = ?$	$10 \times 2 = ?$
$11 - 4 + 6 = ?$	$20 - 7 + 6 = ?$	$13 - 9 + 4 = ?$	$20 - 8 - 4 = ?$
$11 - 8 + 5 = ?$	$20 - 2 - 9 = ?$	$20 - 9 - 4 = ?$	$16 - 7 - 8 = ?$
$17 - 8 + 6 = ?$	$20 - 7 + 6 = ?$	$11 - 6 + 8 = ?$	$19 - 2 - 8 = ?$
$12 - 6 + 7 = ?$	$12 - 7 + 6 = ?$	$20 - 5 - 9 = ?$	$20 - 7 - 5 = ?$
$15 - 6 + 7 = ?$	$14 - 7 + 9 = ?$	$20 - 9 - 8 = ?$	$19 - 8 - 7 = ?$
$20 - 6 + 5 = ?$	$20 - 9 + 4 = ?$	$13 - 5 + 9 = ?$	$18 - 8 - 6 = ?$
$11 - 1 + 8 = ?$	$20 - 6 - 8 = ?$	$18 - 9 + 3 = ?$	$15 - 9 - 3 = ?$
$17 - 9 + 10 = ?$	$19 - 2 + 2 = ?$	$20 - 8 - 7 = ?$	$20 - 3 - 8 = ?$
$14 - 8 + 7 = ?$	$13 - 5 + 8 = ?$	$17 - 9 + 3 = ?$	$18 - 8 - 6 = ?$
$13 - 9 + 11 = ?$	$20 - 9 - 2 = ?$	$20 - 4 - 8 = ?$	$13 - 7 - 2 = ?$
$15 - 6 + 10 = ?$	$18 - 8 + 6 = ?$	$20 - 7 - 3 = ?$	$19 - 8 + 2 = ?$
$19 - 5 + 6 = ?$	$20 - 6 + 4 = ?$	$20 - 8 - 6 = ?$	$18 - 8 - 5 = ?$
$20 - 8 - 3 = ?$	$19 - 7 - 2 = ?$	$15 + 4 - 7 = ?$	$12 + 8 - 5 = ?$

TENS AND ONES.



32. Look at the first picture. How many piles of ten blocks do you see? How many other blocks?

This number is called *twenty-one*, and it is written 21.

How many tens do you see in the second picture? How many ones?

This number is called *thirty-three*, and it is written 33.

How many tens are there in the next picture? How many ones?

This number is called *forty-five*, and it is written 45.

How many tens are there in the last picture? How many ones?

This number is called *fifty-two*, and is written 52.

We have learned to read tens by calling a number expressing 3 tens *thirty*, four tens *forty*, etc.; and when a number expresses tens and ones, we read it by naming the *tens* first and then the *ones*, without any words between them.

We can tell whether a figure stands for *ones* or for *tens* by the place it occupies. If it represents *ones*, it has the *first* place to the *right*, if *tens* the *next to the left*.

Read the following numbers by telling how many tens and how many ones there are.

Thus: 25 is 2 tens and 5 ones, or twenty-five.

87 14 83 96 12 20 21 38 82 29 30 26

99 31 24 76 33 58 16 23 25 67 52 95

68 77 58 74 88 76 39 26 34 51 90 50

Begin with one and add two each time until you reach twenty-one.

Count by threes to 21.

Three feet make 1 yard. If John has 21 feet of string for his kite, how many yards has he?

How many weeks are there in 21 days?

If Kate makes 7 loaves of cake every day for 3 days, and sells on one day 8 loaves and on another day 6, how many has she left?

What is $\frac{1}{2}$ of 21? What is $\frac{1}{3}$ of 21?

If Fred buys 15 papers of one kind and 6 of another, and sells 8 of them, how many has he left?

How much must Jamie pay for 3 balls, if their prices are 6 cents, 9 cents, and 4 cents? How many cents will he have left, if he had 21 cents at first?

Harold worked 10 examples one day, 7 the next day, and 4 the next. How many did he work in the three days? If all but four were correct, how many were correct?

If I have 21 nuts, to how many children can I give 7 each?

If I rode north 6 miles, then east 11 miles, then south 6 miles, how far was I from the starting-place?

If I traveled from home 11 miles on Monday, 7 miles farther on Tuesday, and 3 miles on Wednesday; and on my way back on Thursday walked 10 miles and on Friday 6 miles, how many miles was I then from home?

$$9+2=? \quad 49+2=? \quad 21+7=? \quad 3 \times 3=?$$

$$7+4=? \quad 37+4=? \quad \frac{1}{2} \text{ of } 21=? \quad 4 \times 3=?$$

$$8+3=? \quad 78+3=? \quad \frac{2}{3} \text{ of } 21=? \quad 5 \times 3=?$$

$$19+2=? \quad 79+2=? \quad \frac{1}{4} \text{ of } 21=? \quad 6 \times 3=?$$

$$17+4=? \quad 47+4=? \quad \frac{3}{4} \text{ of } 21=? \quad 7 \times 3=?$$

$$18+3=? \quad 58+3=? \quad \frac{4}{5} \text{ of } 21=? \quad 1 \times 7=?$$

RAPID WORK.

$$8+3+5-7-3+10-8+6-9+1+8=?$$

$$13-6+7-5-2+7+3-10-6+8+7-8+9=?$$

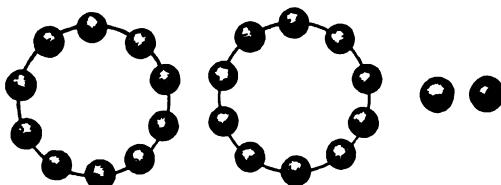
$$11+8-7-6+13-10-7+6+8-9-3+8+9=?$$

$$21-2-7-4-5+6+8-9+4+9-6+7-4=?$$

$$13+5+2-17+6+5-7+9-11+15-13+9-8+4=?$$

$$15-6+7-9+12-11+6-9-2+16-13+15-10=?$$

$$7+8-9+12-11+6-9-2+16-13+15-18+1=?$$



33. Twenty and two are how many?

Count by twos to twenty-two.

What is one half of twenty-two?

How many figures are required to write twenty-two?

For what does the left hand figure stand?

For what does the right hand figure stand?

Donald has 3 pansies, 4 pinks, 5 violets, 6 roses, and 4 lilies. How many flowers has he?

There are 22 children in Fred's class. If 11 stand in a row, how many rows are there?

$$22-2-2-2, \text{ etc., to 0} \quad 18+?=22 \quad 20+?=22$$

$$22-3-3-3, \text{ etc., to 1} \quad 18+?=22 \quad 17+?=22$$

$$22-4-4-4, \text{ etc., to 2} \quad 15+?=22 \quad 14+?=22$$

$$22-5-5-5, \text{ etc., to 2} \quad 19+?=22 \quad 16+?=22$$

How many two-cent postage stamps can I buy for 22 cents? For 20 cents? For 10 cents? For 18 cents?

1 17
 2 15 This is the way Emma adds a column and tests it to
 3 12 see if it is correct.
 1 11 She found the sum to be 18. She then proved her
 2 9 result correct by taking in succession each number
 3 6 from 18, thus: 1 from 18 leaves 17; 2 from 17 leaves
 1 5 15; 3 from 15 leaves 12; 1 from 12 leaves 11; 2 from
 2 3 11 leaves 9, etc. After taking away the last number,
 3 0 0 was left. Test your results in the same way in the
 following examples.

18

Add and test:

3	2	3	1	30	20	3	3	2	1
2	3	3	2	20	30	4	3	2	3
2	2	2	3	10	30	3	3	2	3
2	1	3	2	30	20	4	3	2	3
3	3	1	2	20	10	3	3	2	3
2	2	3	3	10	20	4	3	2	3
2	2	2	3	30	30	3	3	2	3
1	2	3	3	20	30	4	3	2	3
3	4	1	3	10	20	3	3	2	3
—	—	—	—	—	—	—	—	—	—

The sign \$ stands for dollars.

\$4	\$7	\$5	6	8	5	9	4	3	2	1
4	2	5	3	4	2	3	7	1	3	6
4	7	5	6	5	4	2	6	4	4	2
4	2	5	3	3	6	3	3	8	5	8
4	2	0	2	1	3	1	1	2	4	3
1	2	1	2	1	1	1	1	1	4	1
—	—	—	—	—	—	—	—	—	—	—

$$22+2=? \quad 22+11=? \quad \frac{1}{2} \text{ of } 22=? \quad \frac{1}{11} \text{ of } 22=?$$

$$\frac{2}{11} \text{ of } 22=? \quad \frac{8}{11} \text{ of } 22=? \quad \frac{4}{11} \text{ of } 22=? \quad \frac{7}{11} \text{ of } 22=?$$

34. 20 boys were snow-ball-ing and 3 more joined them. How many were there then?

If each boy threw 1 snow-ball, how many snow-balls were thrown?

If all but 3 hit the mark, how many snow-balls hit the mark?

$9 + 4 = ?$	$10 + 4 = ?$	$29 + 4 = ?$	$39 + 4 = ?$
$6 + 7 = ?$	$16 + 7 = ?$	$26 + 7 = ?$	$36 + 7 = ?$
$8 + 5 = ?$	$18 + 5 = ?$	$48 + 5 = ?$	$78 + 5 = ?$
$3 + 10 = ?$	$19 + 4 = ?$	$49 + 4 = ?$	$72 + 9 = ?$

Write the numbers from 20 to 30; from 30 to 40; from 19 to 10; from 50 to 60; from 50 to 40; from 80 to 90.

What is the largest number you can write, using but one figure? What is the smallest?

What is the smallest number you can write, using only two figures?

What is the largest number that can be written with two figures?

Add first by lines and then by columns:

$$\begin{aligned}2+5+3+4+1+2+3+3 &=? \\6+4+3+3+1+3+2+3 &=? \\5+5+5+5+2+2+2+2 &=? \\4+4+4+4+4+1+1+0 &=? \\1+2+4+3+4+3+4+3 &=?\end{aligned}$$

Add, and test your results as before:

2	3	3	2	4	4	6	2	2	3	2	4
2	2	4	9	3	2	7	9	4	4	6	4
8	1	3	1	2	5	2	5	6	3	3	4
4	9	5	4	1	3	3	3	7	4	2	4
7	6	5	6	7	4	5	3	4	3	6	6

If 23 passengers travel by coach and 7 are seated on top, how many are there inside?

Morton earned 15 cents and his uncle gave him 8 cents more. He then bought 3 oranges at 3 cents apiece. How many cents had he left?

If you have 13 cents, how many more cents do you need to buy a book that costs 20 cents?

How many more cents do you need to buy one that costs 23 cents?

Twenty-three children belong to Miss C's class, but only 19 are present. How many are absent?

There were 6 persons in a car, when 7 more were taken in; soon 5 left and 9 got on. How many were there then in the car?

If the car could seat 23 people, how many more could be seated?

If a lady goes to Boston with 23 dollars and spends 16 dollars, how many dollars has she left?

35. Count by 2's to 24.

Count by 3's to 24, then back to 0.

Count by 4's to 24, then back to 0.

How many two-cent postage stamps can I buy for 24 cents?

How many yards are there in 24 feet?

How many gallons are there in 24 quarts?

Twelve things make a dozen. How many dozen buttons has Ethel, if she has 24 buttons?

Mr. B. had 24 eggs. He put them into boxes holding 6 eggs each. How many boxes did he fill?

If the boxes had held 12 eggs, how many boxes would he have used?

Write all the pairs of numbers, which, added together, make 24.

$$24 - 2 - 3 - 4 - 4 - 3 - 6 = ?$$

24 - 2 - 2 - 2, etc., to 0.

24 - 3 - 3 - 3, etc., to 0.

Subtract each number less than 10 from 24 as many times as you can, and write the results on your slate.

$1 \times 4 = ?$	$3 \times 3 = ?$	$12 + 6 = ?$	$24 + 4 = ?$
$2 \times 4 = ?$	$4 \times 3 = ?$	$12 + 2 = ?$	$24 + 8 = ?$
$3 \times 4 = ?$	$5 \times 3 = ?$	$9 + 3 = ?$	$24 + 12 = ?$
$4 \times 4 = ?$	$6 \times 3 = ?$	$18 + 3 = ?$	$24 + 2 = ?$
$5 \times 4 = ?$	$7 \times 3 = ?$	$12 + 4 = ?$	$\frac{1}{2}$ of 24 = ?
$6 \times 4 = ?$	$8 \times 3 = ?$	$16 + 4 = ?$	$\frac{1}{3}$ of 24 = ?
$1 \times 6 = ?$	$1 \times 8 = ?$	$24 + 3 = ?$	$\frac{3}{4}$ of 24 = ?
$2 \times 6 = ?$	$2 \times 8 = ?$	$21 + 3 = ?$	$\frac{1}{4}$ of 24 = ?
$3 \times 6 = ?$	$3 \times 8 = ?$	$8 + 4 = ?$	$\frac{3}{2}$ of 24 = ?
$4 \times 6 = ?$	$24 + 3 = ?$	$8 + 2 = ?$	$\frac{1}{6}$ of 24 = ?
$1 \times 8 = ?$	$18 + 2 = ?$	$15 + 5 = ?$	$\frac{5}{6}$ of 24 = ?
$2 \times 8 = ?$	$24 + 6 = ?$	$12 + 3 = ?$	$\frac{1}{3}$ of 24 = ?

Twelve inches make a foot. How many feet are there in 24 inches?

Harold has 2 dimes and 4 cents. How much money has he? How many bananas, at 6 cents each, can he buy with his money?

If he should prefer to buy oranges at 3 cents, how many could he buy?

If he should give half his money to his brother, how much would each have?

David walked 24 miles in 8 hours. How far did he walk in one hour?

Rachel had 24 postal cards. She used 6 at one time,

5 at another time, and 7 at another time. How many had she left?

Daisy had 2 dozen dolls' cups. She piled them up, placing half a dozen in a pile. How many piles had she?

Harold, Reuben, James, and Burton have 6 cents each, and each one buys a peach for 2 cents. How much more money do they need to buy a ball for 20 cents?

A piece of cloth contained 24 yards; $\frac{1}{3}$ of it was sold. How many yards were sold? How many yards remained unsold?

Thornton bought 2 dozen postal cards. How much did he pay for them? If he gave the clerk 25 cents, how much change should he receive?

Mr. B. worked at his shop from 7 o'clock in the morning until 12 o'clock noon, and from 1 to 4 o'clock in the afternoon, and at home he worked from 7 to 9 o'clock. How many hours did he work?

36. Lucy has two dimes and a half-dime. To how many cents are they equal?

She spent all but 4 cents for 3 paper dolls. How many cents did she spend? What was the cost of each doll?

If Lulu paid 25 cents for a slate and sponge, and the sponge alone cost 6 cents, what was the price of the slate?

A milkman who had 25 quarts of milk put it in gallon (4 quart) cans. How many cans did he fill, and how much milk had he left?

How many people could he supply with 1 quart a day? With 5 quarts? With 3 quarts, and how much would be left?

Harold is 9 years old, and Arthur is 4 years younger. In how many years will Arthur be 25 years old? How many years before Harold will be 25?

Daisy had 25 acorns, which she placed in 5 equal piles. How many did she put in each pile?

What is $\frac{1}{5}$ of 25 cents? $\frac{1}{5}$ of 25 children? $\frac{1}{5}$ of 25 hens? $\frac{2}{5}$ of 25 trees?

Lena had a quarter of a dollar (25 cents), and spent 8 cents. How much had she left?

The cook used 2 dozen eggs for the Christmas dinner. If she had 25 eggs at first, how many had she left?

How many half-dimes can you get in change for a quarter of a dollar?

$$24 + 1 = ?$$

$$25 - 4 - 4 - 4, \text{ etc.}$$

$$25 - 2 - 2, \text{ etc.}$$

$$25 - 5 - 5 - 5, \text{ etc.}$$

$$25 - 3 - 3 - 3, \text{ etc.}$$

$$25 - 6 - 6 - 6, \text{ etc.}$$

$$1 \times 5 = ? \quad 6 \times 5 = ? \quad 25 + 1 = ? \quad 20 + ? = 25 \quad ? + 16 = 25$$

$$2 \times 5 = ? \quad 7 \times 5 = ? \quad \frac{1}{5} \text{ of } 25 = ? \quad 15 + ? = 25 \quad ? + 14 = 25$$

$$3 \times 5 = ? \quad 8 \times 5 = ? \quad \frac{2}{5} \text{ of } 25 = ? \quad 23 + ? = 25 \quad ? + 21 = 25$$

$$4 \times 5 = ? \quad 25 \div 5 = ? \quad \frac{3}{5} \text{ of } 25 = ? \quad 5 + ? = 25 \quad ? + 11 = 25$$

$$5 \times 5 = ? \quad 25 \div 10 = ? \quad \frac{4}{5} \text{ of } 25 = ? \quad 7 + ? = 25 \quad ? + 18 = 25$$

37. Two tens and six are how many?

If I have 26 pictures and give away one half of them, how many do I give away? How many have I left?

$$2 \times ? = 26$$

$$26 - 13 = ?$$

Write all the pairs of numbers you can whose sum equals 26.

Agnes bought 26 buttons at the store. The buttons were on cards, having 12 buttons each. How many cards did she buy, and how many buttons besides?

In our school there are 23 pupils. On Saturday 5 boys and 3 girls went skating, 6 girls and 4 boys went coasting, and the rest went to Boston. How many pupils went to Boston?

James solved 7 examples on Monday, 8 on Tuesday, and 6 on Wednesday. How many did he solve in all?

A pole stands 7 feet in the mud, 6 feet in the water, and 9 feet above water. How long is the pole?

I counted the birds on 4 telegraph wires. On the first wire there were 6 birds, on the second, 5, on the third, 6, and on the fourth, 6. How many birds did I count?

A boy had 6 copper cents, 2 half-dimes, and a dime. How much money had he?

Ethel has five half-dimes and a cent. How much money has she?

How many Christmas cards, at 4 cents each, can Ethel buy with her money, and how many cents will she have left?

Frank is 26 years old, and Philip is 19. What is the difference in their ages?

A newsboy buys 16 Heralds and 10 Transcripts. He sells 12 Heralds and 7 Transcripts. How many papers has he yet on hand?

A man had 26 gallons of vinegar. He first sold one half of it, and later he sold 4 gallons more. How many gallons did he sell? How many had he left?

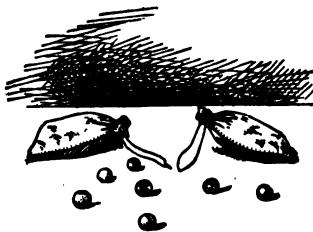
A boy had 12 slate pencils and 11 lead pencils; he gave away half the slate pencils. How many pencils had he left?

Count by 2's to 26.

Write the table of 2's to 26, thus:

$$1 \times 2 = ? \quad 2 \times 2 = ? \quad 3 \times 2 = ? \text{ etc.}$$

38. If there are 10 marbles in each of the bags in the picture, how many marbles are there in all?



Two tens and 7 ones are how many?

Two dimes and 7 cents are how many cents?

Fred had 26 apples in his basket, and 1 in his hand. How many had he?

To how many children can he give 3 apples each? 9 each? 5 each, and how many will be left?

How many yards are there in 27 feet?

If I divide 27 pieces of candy equally among 3 children, how many pieces will each have?

What is one third of 27? Two thirds of 27?

A grocer having 27 barrels of flour, put them in 9 equal piles. How many barrels were there in each pile?

What is one ninth of 27 melons? Two ninths?

If I have $\frac{2}{3}$ of 27 dollars, how many dollars have I?

A man having 27 cows sold $\frac{2}{3}$ of them. How many did he sell? How many had he left?

A fruit dealer found that $\frac{1}{3}$ of his 27 baskets of peaches was spoiled. How many were spoiled? How many baskets of good peaches had he left?

$$4 \times 3 = ? \quad 3 \times 3 = ? \quad 4 \times 7 = ? \quad 6 + 3 = ? \quad \frac{1}{3} \text{ of } 27 = ?$$

$$1 \times 3 = ? \quad 7 \times 3 = ? \quad 27 + 3 = ? \quad 24 + 3 = ? \quad \frac{1}{3} \text{ of } 21 = ?$$

$$6 \times 3 = ? \quad 5 \times 3 = ? \quad 27 + 9 = ? \quad 24 + 8 = ? \quad \frac{1}{3} \text{ of } 15 = ?$$

$$2 \times 3 = ? \quad 1 \times 9 = ? \quad 18 + 3 = ? \quad 18 + 8 = ? \quad \frac{1}{3} \text{ of } 27 = ?$$

$$8 \times 3 = ? \quad 2 \times 9 = ? \quad 12 + 3 = ? \quad 8) 9(\quad 18 + 2 = ?$$

$$9 \times 3 = ? \quad 8 \times 9 = ? \quad 8) 8(\quad 7) 21(\quad 18 - 9 = ?$$

Add by lines and columns:

$$\begin{array}{ll}
 3+3+3+3+3+3+3+3=? & 4+8+4+7+4+1=? \\
 4+2+5+7+1+6+0+2=? & 5+4+5+3+4+6=? \\
 8+4+1+9+0+2+2+2=? & 5+3+1+7+5+4=? \\
 6+6+3+4+4+2+1+5=? & 7+2+8+6+3+3=? \\
 7+8+2+5+4+5+4+1=? & 6+4+2+4+4+7=? \\
 \hline
 \end{array}$$

$$27+1=?$$

Count by 2's to 28.

Count by 4's to 28.

Count by 7's to 28.

How many tens and how many ones are there in 28?

If Nellie puts 28 peaches on four plates, putting the same number on each plate, how many will there be on each plate?

If she gives away the peaches on one plate, what part of the whole will she give away? What is $\frac{1}{4}$ of 28?

Max bought 7 pairs of shoes for 28 dollars. What was the cost of 1 pair?

What is $\frac{1}{7}$ of 28? $\frac{2}{7}$ of 28? $\frac{3}{7}$ of 28?

How many five-cent pieces and how many cents will make 28 cents?

Fred was sick for 28 days. How many weeks was that?

Mr. Brown had 28 pecks of potatoes, which he sold by the bushel (4 pecks). How many bushels did he sell?

28 pints equal how many quarts?

28 quarts equal how many gallons?

William spent one half of his 28 cents for a slate and 6 cents for a lead pencil. How much did he spend? How much had he left?

$$28-5-9-7-8-4=?$$

$$28-8-6-2-2-1=?$$

$$28-9-2-8-1-7=?$$

$$28-4-4-4-4-4=?$$

$$28-7-7-7-7=?$$

Add and prove by subtracting as on page 59.

4	5	6	2	1	8	4
5	6	7	3	2	4	4
6	2	8	4	3	8	4
4	9	4	9	4	4	4
7	4	0	7	5	8	4
2	1	8	4	6	4	4
—	—	—	—	—	—	—

$$28+1=?$$

Beginning with 1 count by 2's to 29.

Beginning with 1 count by 4's to 29.

Beginning with 1 count by 7's to 29.

39. After giving 9 cents for some candy, Anita had 2 dimes remaining. How many cents did she have at first?

At 9 cents a spool, how much will 3 spools of silk cost?

One day Alice worked 4 hours at 7 cents an hour, and gave 10 cents for car fare. How many cents had she left?

Henry had 29 cents, but spent 10 cents for a top. How much had he left? How much did he lack of having a quarter of a dollar?

A boy who had 9 cents went on 3 errands, earning 5 cents for each. How much money did he have then?

An organ grinder received at one house 10 cents, at

another 10 cents, and at another 9 cents. How much did he receive from them all?

Henry caught 15 fishes and Oren 14. How many did both catch?

After spending 19 cents for fishing tackle, a boy had 10 cents left. How much had he at first?

Mary went to school 18 days in January and 11 in February. How many days did she attend in those months?

$$29 - 5 - 4 - 3 - 6 = ?$$

$$29 - 7 - 3 - 4 - 2 = ?$$

$$29 - 8 - 8 - 8 - 3 = ?$$

$$29 - 5 - 6 - 4 - 9 = ?$$

$$29 - 8 - 3 - 9 - 9 = ?$$

40. Morton paid 3 dimes for a picture-book. What did it cost?

Lulu had 3 dimes, and spent one half her money. How many cents did she spend?

Susan's clothes-line is 10 yards long. How many feet long is it?

How many half-dimes can I get in exchange for 30 cents?

How many oranges, at 5 cents apiece, can be bought for 3 dimes?

Lena bought half a dozen bananas at 5 cents each. What did they cost?

Lucy has 25 cents and a half-dime. How many quarts of cherries, at 6 cents a quart, can she buy with the money?

Clarence lives 6 blocks from the schoolhouse. How many blocks does he walk to and from school each day, if he goes home to dinner?

Some girls wanted to buy a game that cost 30 cents; 3 gave 2 cents each; 4 gave 3 cents each; 2 gave 4 cents each. How many girls contributed? How much more do they need to pay for the game?

Lella bought 3 pounds of sugar at 7 cents a pound, and gave the storekeeper 3 dimes. What change should she receive?

$$10 + ? = 30$$

$$18 + ? = 30$$

$$14 + ? = 30$$

$$17 + ? = 30$$

$$15 + ? = 30$$

$$12 + ? = 30$$

$$20 + ? = 30$$

$$19 + ? = 30$$

$$11 + ? = 30$$

$$25 + ? = 30$$

$$16 + ? = 30$$

$$30 - 8 = ?$$

From 30 take 2 as many times as you can, naming results; as, 28, 26, etc.

In the same way subtract 8, 5, 6, 7, 4, 8.

Anson had 30 rabbits, but 10 of them got away. How many had he left?

Mary had 2 dimes and a five-cent piece. After paying 10 cents for a composition book and 5 cents for a pencil, how much had she left?

Hannah attended school every school day for 30 days, but her sister Sarah was absent 8 days during this time. How many days did Sarah attend?

A month is commonly regarded as 30 days. How many days is half a month?

The fare from Andes to Rayfield is 15 cents. A pupil's ticket from Andes to Rayfield and back is only 18 cents. How much cheaper is the pupil's ticket than the regular fare?

A boy who had 3 dimes paid 25 cents for a ticket to a museum. How much money had he left?

REVIEW EXERCISES.

41. Answers to be given instantly.

$\frac{1}{2}$ of 10=?	16 - 9=?	20 - 5=?	19 - 7=?
$\frac{1}{2}$ of 20=?	15 + 3=?	24 + 4=?	3 3's=?
7×2 ?=?	15 - 8=?	30 + 10=?	12 + 4=?
30 + 5=?	25 - 8=?	6 + 7=?	18 + 6=?
9 + 5=?	3 10's=?	11 - 9=?	2 \times 8=?
$\frac{1}{3}$ of 27=?	4) 28(4 + 7=?	12 2's=?
$\frac{3}{4}$ of 27=?	28 + 7=?	8 - 3=?	$\frac{1}{6}$ of 30=?
30 + 3=?	13 - 9=?	18 - 3=?	8 2's=?
28 + 7=?	11 - 2=?	3 \times 7=?	21 - 3=?
$\frac{1}{4}$ of 28=?	9 + 9=?	13 - 7=?	7 \times 2=?
13 - 8=?	19 + 9=?	9 + 8=?	6 \times 5=?
3 \times 9=?	11 - 6=?	19 + 8=?	4 \times 6=?
9 \times 3=?	5 + 7=?	5 \times 5=?	24 + 3=?
12 - 5=?	17 - 8=?	$\frac{1}{5}$ of 25=?	$\frac{1}{2}$ of 16=?
$\frac{1}{5}$ of 24=?	3 \times 6=?	$\frac{1}{10}$ of 20=?	$\frac{1}{3}$ of 18=?
$\frac{3}{4}$ of 24=?	9 \times 3=?	$\frac{1}{4}$ of 21=?	$\frac{1}{10}$ of 20=?
24 + 12=?	7 \times 3=?	12 + 6=?	4 \times 8=?
24 + 8=?	18 + 3=?	24 + 6=?	8 \times 2=?
9 - 5=?	8 3's=?	3 \times 5=?	22 + 2=?
3 \times 4=?	16 - 5=?	$\frac{1}{3}$ of 21=?	7 \times 3=?
3 \times 8=?	12 - 8=?	$\frac{1}{3}$ of 21=?	21 + 7=?
$\frac{1}{4}$ of 16=?	7 + 8=?	13 - 4=?	$\frac{1}{5}$ of 16=?
$\frac{3}{4}$ of 16=?	4 3's=?	$\frac{1}{4}$ of 24=?	$\frac{3}{5}$ of 16=?
2 \times 12=?	14 - 9=?	5 3's=?	6 3's=?
9 - 5=?	19 + 3=?	15 - 8=?	8 + 7=?

$19 - 5 = ?$	$25 + 5 = ?$	$4 \ 4's = ?$	$2 + 9 = ?$
$29 - 5 = ?$	$7 \ 4's = ?$	$6 \ 4's = ?$	$11 \ 2's = ?$
$8) 24 ($	$12 - 9 = ?$	$3 \ 6's = ?$	$4 \times 7 = ?$
$11 \ 2's = ?$	$17 - 8 = ?$	$\frac{1}{6} \text{ of } 24 = ?$	$\frac{1}{8} \text{ of } 24 = ?$
$2 \times 9 = ?$	$30 + 5 = ?$	$5 + 9 = ?$	$\frac{5}{8} \text{ of } 24 = ?$
$3 \times 9 = ?$	$23 + 7 = ?$	$8 + 5 = ?$	$18 + 2 = ?$
$9) 27 ($	$3 \times 6 = ?$	$13 + 5 = ?$	$\frac{1}{3} \text{ of } 30 = ?$
$18 + 9 = ?$	$19 - 8 = ?$	$28 + 4 = ?$	$2 \times 9 = ?$
$18 - 9 = ?$	$29 - 8 = ?$	$8 \ 3's = ?$	$22 + 11 = ?$
$5 \ 6's = ?$	$20 + 4 = ?$	$2 \ 8's = ?$	$\frac{1}{3} \text{ of } 9 = ?$

NUMBERS FROM 30 TO 100.

42. Count by 4's to 32.

Write the table of 4's, thus:

$$1 \times 4 = ? \quad 2 \times 4 = ? \quad 3 \times 4 = ? \text{ etc.}$$

There are 4 quarts in a gallon. How many quarts are there in 2 gallons? In 4 gallons? In 7 gallons? In 8 gallons?

There were 4 sleds on Morton's hill last Saturday. If each one held 8 children, how many children were coasting on the hill?

$$1 \times 8 = ? \quad 2 \times 8 = ? \quad 3 \times 8 = ? \quad 4 \times 8 = ?$$

Bertie had 32 cents. If he gave Helen $\frac{1}{4}$ of his money, how many cents did he give her?

Mr. Young had 32 sheep, but he sold $\frac{1}{8}$ of them. How many did he sell? How many had he left?

John had 30 cents; he spent one half of it for a slate, and 2 cents for a sponge. How much had he left? Then, if his mother gave him 10 cents, how much had he?

How many pecks (8 quarts) are there in 32 quarts?

How many 3-cent pieces can I get for 8 dimes and 8 cents?

Write the table of 3's, thus:

$$1 \times 3 = ? \quad 2 \times 3 = ? \quad 3 \times 3 = ? \text{ etc.}$$

$$1 \times 11 = ? \quad 2 \times 11 = ? \quad 3 \times 11 = ?$$

Julia had 33 cents, and Olive $\frac{1}{3}$ as many. How many cents had Olive?

$$33 \div 3 = ? \quad 33 \div 11 = ? \quad \frac{1}{3} \text{ of } 33 = ? \quad \frac{1}{11} \text{ of } 33 = ?$$

A walk measured 11 yards in length. Since a yard is 3 feet long, how many feet long was the walk?

I spent 33 cents for balls at 11 cents apiece. How many balls did I buy?

Count by 2's to 34, and back from 34 to 0.

$$17 + 17 = ? \quad 2 \times 17 = ? \quad 34 + 2 = ?$$

Two men start from the same place and travel in opposite directions. One travels at the rate of 3 miles an hour, the other at the rate of 2 miles an hour. How far apart are they at the end of 2 hours? 5 hours? 7 hours? 8 hours? 6 hours?

Count by 5's to 60. Count by 7's to 70. Count by 10's to 70.

Donna worked 7 examples each day for 5 days. How many did she work?

Mrs. H. bought from the milkman 35 quarts of milk in one week. What was the average number of quarts she bought per day?

How many days are there in 2 weeks? In 5 weeks? In 4 weeks? In 3 weeks? In 6 weeks? In 8 weeks? In 7 weeks?

If 5 men can do a piece of work in 7 days, how long will it take 1 man to do it?

How many weeks are there in 35 days? In 14 days? In 28 days? In 42 days?

$$\begin{array}{lll} \frac{1}{5} \text{ of } 35 = ? & \frac{1}{7} \text{ of } 35 = ? & 35 + 7 = ? \\ \frac{1}{7} \text{ of } 35 = ? & \frac{1}{5} \text{ of } 35 = ? & 35 + 5 = ? \end{array}$$

Two men start from the same place and travel in opposite directions. One travels 9 miles an hour, the other travels 7 miles an hour. How far apart will they be in 2 hours?

Suppose the men traveled in the *same* direction, how far apart would they be in 2 hours? In 7 hours? In 11 hours? In 12 hours?

Harry earned 7 half-dimes while Lucy earned 3 dimes and a five-cent piece. Which earned the larger sum of money?

At 7 cents a yard, what is the cost of 4 yards of cambric? Of 3 yards? Of 5 yards? Of 7 yards? Of 6 yards?

How many weeks was Lulu away at the seashore, if she was gone 21 days? 14 days? 28 days? 42 days? 63 days? 49 days? 56 days?

$$\begin{array}{llllll} 1 \times 6 = ? & 2 \times 6 = ? & 3 \times 6 = ? & 4 \times 6 = ? & 5 \times 6 = ? & 6 \times 6 = ? \\ 7 \times 6 = ? & 8 \times 6 = ? & 9 \times 6 = ? & 10 \times 6 = ? & 11 \times 6 = ? & 12 \times 6 = ? \end{array}$$

Emily is 18 years old, and her uncle is twice as old. How old is he?

What is $\frac{1}{2}$ of \$36? $\frac{1}{2}$ of 36 men? $\frac{1}{2}$ of 36 bushels?

How many times is 4 contained in 12? In 24? In 36? In 20? In 16? In 8? In 28? In 32?

There are 6 working days in a week. How many working days are there in 2 weeks? In 6 weeks? In 8

weeks? In 5 weeks? In 4 weeks? In 7 weeks? In 10 weeks? In 8 weeks? In 9 weeks? In 11 weeks? In 12 weeks?

$$\begin{array}{llll}
 12+6=? & 24+6=? & 30+6=? & 36+6=? \\
 24+6=? & 66+6=? & 60+6=? & 72+6=? \\
 6 \times 6=? \times 3 & 6 \times 6=? \times 9 & 6 \times 6=? \times 12 & 5 \times 6=? \times 8 \\
 2 \times 10=? \times 4 & 4 \times 4=2 \times ? & 6 \times 2=? \times 4 & 4 \times 4=? \times 8 \\
 48+6=? & 18+6=? & 42+6=?
 \end{array}$$

How many yards of cloth, at \$3 a yard, will pay for 5 barrels of flour at \$6 per barrel?

Willard sold 12 barrels of apples at \$3 per barrel, and spent half the money for books at \$2 each. How many books did he buy?

Otto sold 6 quarts of berries at 6 cents a quart, and spent the money for 9 bananas. How much did he pay apiece for the bananas?

There are 9 children in the drawing class, and 4 times as many in the arithmetic class. How many are there in the arithmetic class?

At 4 cents a yard, what is the cost of 2 yards of ribbon? Of 4 yards? Of 6 yards? Of 9 yards? Of 10 yards? Of 8 yards? Of 3 yards?

How many weeks' work are there in 36 working days? In 18? In 24? In 30? In 12? In 42? In 60? In 72?

Count by 2's to 38.

$$9+9=? \quad 19+9=? \quad 29+9=? \text{ etc.} \quad 19+19=? \text{ etc.}$$

Count by 3's to 39. Count by 4's to 48.

At 4 cents apiece, what must I pay for 5 oranges? For 3? For 8? For 6? For 10? For 4? For 7? For 9? For 11? For 1 dozen?

Bert rode his pony 8 miles a day for 5 days. How far did he ride?

How many eggs, at 4 cents each, will pay for 5 yards of ribbon at 8 cents a yard?

If a stage goes 8 miles an hour, how many hours will it take to travel 24 miles? 40 miles? 16 miles? 48 miles? 32 miles? 56 miles? 72 miles? 64 miles?

What is $\frac{1}{2}$ of 40? $\frac{1}{3}$ of 40? $\frac{2}{3}$ of 40? $\frac{1}{4}$ of 40?
 $\frac{1}{10}$ of 40? $\frac{8}{10}$ of 40? $\frac{1}{4}$ of 40? $\frac{2}{3}$ of 40?

How many five-cent loaves of bread can Florence buy with 25 cents? With 35 cents? With 40 cents? With 50 cents?

How many gallons are there in 12 quarts? In 28 quarts? In 36 quarts? In 40 quarts? In 48 quarts? In 16 quarts?

How many quarts are there in 16 pints? In 24 pints? In 20 pints? In 8 pints? In 40 pints? In 38 pints?

How many pecks are there in 3 bushels? In 8 bushels? In 10 bushels? In 6 bushels? In 4 bushels? In 9 bushels? In 7 bushels?

If one pencil costs 5 cents, what is the cost of 3 pencils? Of 5 pencils? Of 4 pencils? Of 2 pencils? Of 6 pencils? Of 7 pencils?

At 7 cents a quart, what is the cost of 4 quarts of blueberries? Of 6 quarts? Of 3 quarts? Of 2 quarts? Of 7 quarts?

Count by 7's to 70. Count by 8's to 80.

Lucy bought 6 paper dolls at 7 cents each. She gave the clerk 50 cents. What change should she receive?
 $6 \times 7 = 2 \times ?$

Donna has 9 half-dimes in her pocket book. To how many cents are they equal?

How many toy books, at 6 cents each, can she buy, and how many cents will she have left?

If Clarence sleeps 8 hours at night, how many hours does he sleep in 3 nights? In 5 nights? In 7 nights? In 4 nights? In 6 nights?

Helen had 3 five-cent pieces, and Nannie had 3 dimes. How many sheets of drawing paper, at 9 cents each, can they buy if they put their money together?

Count by 5's to 60. Count by 9's to 90.

How many 5's are there in 25? In 40? In 15? In 45? In 30? In 50? In 35? In 60? In 20? In 55?

How many 9's are there in 18? In 45? In 27? In 36? In 54? etc.

If a boy can buy 9 marbles for a cent, how many marbles can he buy for 3 cents? For 2 cents? For 5 cents? For 4 cents? For 6 cents? For 7 cents?

Bert's pony can travel 8 miles an hour. How much farther can he travel in 6 hours than Harold's pony, which can travel only 6 miles an hour?

$$(9 \times 5) - (5 \times 6) = ? \quad (4 \times 10) - (4 \times 8) = ?$$

$$(4 \times 9) - (3 \times 4) = ? \quad (5 \times 10) - (6 \times 6) = ?$$

$$(7 \times 7) - (6 \times 7) = ? \quad (7 \times 4) - (9 \times 3) = ?$$

$$(6 \times 5) - (3 \times 5) = ? \quad (8 \times 6) - (5 \times 4) = ?$$

$$(4 \times 10) - (5 \times 7) = ? \quad (5 \times 5) - (3 \times 5) = ?$$

$$(4 \times 4) + (5 \times 2) = ? \quad (12 \times 3) - (4 \times 9) = ?$$

$$(6 \times 7) - (3 \times 7) = ? \quad (6 \times 4) + (3 \times 5) = ?$$

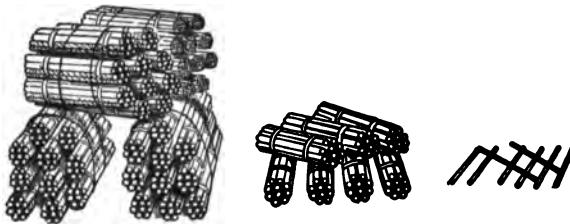
$$(5 \times 8) - (3 \times 3) = ? \quad (3 \times 6) + (3 \times 4) = ?$$

$$(3 \times 11) + (2 \times 5) = ? \quad (8 \times 4) - (4 \times 7) = ?$$

How many 10's are there in 30? In 50? In 90? In 100?

What is $\frac{1}{2}$ of 50 cents? Of 40 cents? Of 30 cents?

NOTATION AND NUMERATION.



43. We have learned that ten ones make one ten. Now take ten tens and tie them together. Ten tens are called *one hundred*.

In writing hundreds, the *figure* representing *hundreds* is placed in the *third* place from the right.

Thus, one hundred is written 100.

44. Write with figures :

Two hundred.	Three hundred.	Nine hundred.
Seven hundred.	Four hundred.	Six hundred.
Five hundred.	Eight hundred.	One hundred.

45. When numbers are expressed by three figures, the figure at the right represents *ones* or *units*, the second figure *tens*, and the third *hundreds*.

46. In reading numbers expressed by three *figures*, the tens are read after the hundreds without the word *and*.

Thus, 478 is read four hundred seventy-eight, instead of four hundred *and* seventy-eight. 205 is read two hundred five, not two hundred *and* five.

EXERCISES.

47. What number is represented in the picture on page 78?

Show with objects, and tell what you understand by

303 121 763 286 431 295 333 107 113 103
 400 200 101 111 321 117 115 240 276 189

Write the following numbers, placing units under units, tens under tens, etc.:

Three hundred forty-six.	Sixteen.
Five hundred eighteen.	Six hundred eighty-five.
Ninety-two.	Forty-seven.
One hundred seventy-five.	Seven hundred sixteen.
Two hundred eighty.	Two hundred two.
Seven hundred eleven.	Four hundred eleven.
Four hundred forty-four.	Eight hundred.
Three hundred five.	Five hundred twenty-nine.
Eight hundred seventeen.	Nine hundred seventy-three.
Six hundred nine.	Seven hundred sixty-four.

Read the numbers you have written.

How many figures are needed to express hundreds?

48. The method of expressing numbers by figures or letters is called **Notation**.

49. The method of expressing numbers by figures is called the **Arabic Notation**.

It is called the Arabic Notation because it was first introduced into Europe by the Arabs.

50. The method of reading numbers is called **Numeration**.

EXERCISES.

51. 1. Write in columns of ten numbers each all the numbers from one hundred to one hundred forty-nine.

2. From two hundred fifty-one to three hundred.

3. What number is next greater than 549, 328, 191, 400, 589, 240, 379, 599, 109, 765?

4. Make the following numbers larger by one hundred, and write them : 17, 89, 30, 4, 52, 305.
 5. What number comes next before 100, 700, 201, 870, 310 ?
 6. How many hundreds are there in each of the following numbers ? How many tens ? How many units ?

468 453 754 902 999 837 964 815
 125 602 882 176 731 395 618 274

52. What is the smallest number you can express by three figures ? The largest ?

999 If we add 1 to 999, we have nine units and 1,
 1 making ten units or one ten. Adding the one ten and
 1000 the nine tens, we have ten tens or one hundred. Adding
 the one hundred to the nine hundreds, we have
 ten hundreds, which we call by the new name one thousand.

7000 is read, seven thousand. 1720 is read, one thousand seven hundred twenty.

EXERCISES.

Read the following :

1.	2.	3.	4.	5.	6.	7.
2000	3000	5000	7000	4000	8000	9000
3791	4809	7002	9100	4700	2083	1005
4263	5074	2380	1234	5678	7985	9090

8. What place, counting from the right, do thousands occupy ?

9. What is the smallest whole number that can be expressed by four figures ? What is the greatest ?

10. How many units, tens, and hundreds are expressed by each of the following numbers ?

Write and read :

11.	12.	13.	14.	15.	16.	17.
6709	7891	7000	7777	1098	5055	9100
7253	8005	7700	4028	7005	7643	1001
1389	6666	7770	2003	4809	5010	1110

18. Read the numbers in 11, 12, 13, and 14, reading the thousands and hundreds together as hundreds.

Thus, 6709, or sixty-seven hundred nine.

19. In 8888, for what does each of the 8's stand?

20. What effect would it have upon the value of 7 in 71, if a cipher were placed to the right of the 1?

53. When we reach a thousand, we begin to count the thousands as we did the units, thus: 1 thousand, 2 thousand, etc., up to 999 thousand, and when we have a thousand thousand, we call the number **one million**.

A thousand million we call a **billion**.

A thousand billion we call a **trillion**.

54. For convenience, when more than three figures are employed to express any whole number, they are divided into groups called **Periods**, the first of which, counting from the right, is used to denote any number from 1 to 999 *units*; the second, from 1 to 999 *thousands*; the third, from 1 to 999 *millions*, etc.

For convenience in reading, the periods are usually separated from each other by commas.

TABLE.

Hundreds of Tens of Units of TRILLIONS.	Hundreds of Tens of Units of BILLIONS.	Hundreds of Tens of Units of MILLIONS.	Hundreds of Tens of Units of THOUSANDS.	Hundreds of Tens of Units of UNITS.
7 4 3,	9 8 6,	5 4 3,	2 0 6,	7 1 9
2 0 3,	7 7 7,	4 5 0,	8 1 0,	2 0 3
		2 5,	6 8 7,	1 0 0,
				7 2 1
4,	4 0 0,	4 0 0,	4 9 6,	9 0 0
4 6,	7 8 5,	0 0 0,	0 1 0,	0 0 3
7 9 6,	0 4 5,	8 7 0,	9 0 3,	4 2 9

The first number in the table is read: *seven hundred forty-three trillion, nine hundred eighty-six billion, five hundred forty-three million, two hundred six thousand, seven hundred nineteen.*

Read the foregoing numbers, looking at the headings until you become familiar with the names of the periods.

55. *When we read numbers, we begin at the right and separate the numbers into periods of three figures each. Then, beginning at the left, we read each period as if it stood alone, adding its name.*

Each period, except the highest or left-hand period, must contain three figures.

EXERCISES.

56. Write and read:

1.	2.	3.	4.
700,000	903,744	300,300	17,019
965,237	200,080	37,961	586,240
83,016	147,906	145,808	770,300
728,000	400,004	307,029	562,999
5.			
1,789,645	769,834,521	211,467,431	
29,783,240	888,971,646	462,293,070	

8. Erase the figure at the right hand of each number in 5, 6, and 7, arrange the remaining figures in periods anew, by placing commas where they belong, and then read the numbers.

9. Continue thus with the given numbers, erasing the figures one by one, pointing off into periods correctly, and reading the numbers.

10. Make the following numbers larger by one thousand. By three thousand. By seven thousand. By nine thousand.

11. What number is next greater than 1000? 1699?
2019? 5082? 7200? 1239? 7379? 6299? 9999?
7676? 8109? 4099?

12. What number comes next before 3000? 6000? 5400?
2040? 2010? 5790? 1000? 2010? 9801?

13. How many hundreds and tens are there in 34 tens?
In 86 tens? In 37 tens? In 42 tens? In 128 tens? In
564 tens?

14. How many hundreds are there in 386 units? In 864
units? In 130 units? In 175 units? In 679 units? In
534 tens? In 643 tens? In 161 tens? In 879 tens?

15. How many thousands and hundreds are there in 75
hundreds? In 51 hundreds? In 42 hundreds? In 78 hun-
dreds? How many in 250 tens? In 175 tens? In 549
tens? In 543 tens? How many in 9875 units?

16. Make the following numbers larger, first by ten thou-
sand; then by fifty thousand; then by seventy thousand:

834 227 386 700 652 200 507 993

17. What number is next greater than 15,999? 150,109?
199,999? 777,779? 976,190? 275,300? 789,999?

18. What number is next less than 400,001? 600,000?
289,223? 500,000? 943,000? 727,800? 482,176?
100,000? 400,680? 400,999? 700,010? 300,000?

WRITTEN EXERCISES.

57. Write in figures:

1. Six thousand, eight hundred forty.
2. Eight thousand, two hundred fifty-seven.
3. Nineteen thousand, five hundred twenty-seven.
4. Eighty-four thousand, six hundred thirty-one.
5. One hundred thousand, seven hundred ten.
6. Fifty-eight thousand, six hundred one.
7. Two hundred thousand, four hundred seventy-five.
8. Four hundred eighteen thousand, two hundred seven.

9. Five hundred seven thousand, thirty.
10. Seven hundred fifteen thousand, nine hundred twenty-four.
11. Thirty-two million, four hundred fifteen thousand, two hundred thirty-three.
12. One hundred six million, eight hundred nine thousand, seven hundred fifty.
13. Two hundred sixty-five million, fifteen thousand, one hundred seventy-nine.
14. Three hundred sixty-five million, forty-nine.
15. Nine million, nine thousand, nine.
16. Eighty million, eighty thousand, eighty.
17. Six hundred million, six hundred thousand, six hundred.
18. Thirty-six million, eighteen thousand.
19. Twenty-four million, eight thousand, eight.
20. Two hundred million, two thousand, two.
21. Thirty-nine billion, sixty million, sixty thousand, sixty.
22. Fifty-two billion, one million, one thousand, one.
23. Sixty billion, sixty million, sixty thousand, sixty.
24. Eight hundred million, eighty-four thousand.
25. Seventy-seven billion, seventy-seven million, seventy-seven thousand, seventy-seven.
26. Four hundred twenty-six million, four hundred thousand, twenty-five.
27. One hundred seventy million, seven.
28. Sixteen hundred seventy-eight.
29. One trillion, seventeen million.
30. Four hundred fifty-six million, ten.
31. Five hundred nine million, four hundred thousand, twenty-five.
32. Nineteen hundred two.
33. One billion, six hundred thousand, four.

NOTATION AND NUMERATION OF UNITED STATES MONEY.

58. Write the names of all the United States coins you have seen, and tell of what each is made. What paper money have you seen?

In the currency of the United States,

10 mills make 1 cent.
100 cents make 1 dollar.

59. The Sign of Dollars is \$. It is written before the number.

Thus, \$27 is read, twenty-seven dollars.

60. In writing *cents* and *mills*, a period called the Decimal Point is placed before the number of cents.

61. *Cents* occupy the *first two* places at the right of the decimal point, and *mills* the *third* place.

Thus, \$.35 is read, thirty-five cents ; \$2.875 is read, two dollars eighty-seven cents five mills.

Mills are not coined. In business five mills or more are called one cent, and less than five are disregarded.

62. If the number of cents is less than ten, a cipher must be written in the first place at the right of the decimal point.

Thus, five cents is written \$.05 ; three cents, \$.03 ; seven dollars two cents five mills is written \$7.025.

EXERCISES.

63. Read the following :

1.	2.	3.	4.	5.
\$1.00	\$1.75	\$1.65	\$3.185	\$0.25.
\$4.87	\$18.05	\$15.43	\$70.10	\$1.05.
\$3.04	\$24.16	\$100.50	\$9.125	\$.40.
\$7.	\$2.357	\$10.	\$0.375	\$7.20.

Write the following in columns, keeping decimal points in a vertical line:

1. Eighteen dollars twenty-four cents.
2. Twenty-six dollars thirty-six cents.
3. Fifty-one dollars five cents five mills.
4. Thirty-eight dollars sixteen cents three mills.
5. Thirty-nine dollars nine cents.
6. Thirty dollars five cents.
7. One hundred dollars ten cents four mills.
8. Six dollars six cents.
9. Write one cent, two cents, three cents, etc., to fifty cents.

10. How many cents are there in \$1? In \$2? In \$3?
In \$4? In \$8? In \$5? In \$9? In \$7? In \$6?

11. Read as cents: \$2; \$2.02; \$3; \$3.05; \$3.75;
\$4.17; \$9.08; \$11.11.

12. Write the following as cents, using the character ¢ for cents:

\$1.25	\$1.10	\$3.75	\$7.45	\$9.67	\$8.80
\$7.05	\$9.81	\$4.13	\$4.18	\$5.56	\$6.72

13. Read the following as dollars and cents:

Thus, 1728¢ = 17 hundred 28 cents, or 17 dollars 28 cents.

2375¢	300¢	1015¢	1005¢	680¢	2793¢
1390¢	970¢	700¢	57345¢	3250¢	1809¢

14. Write the same, using the sign of dollars.

15. How many mills are there in 2¢? In 5¢? In 7¢?
In 9¢? In 10¢? In 70¢? In 30¢? In 90¢? In 15¢?

16. How many mills are there in 100¢? In 300¢? In
\$1? In \$2? In \$9? In \$1.20? In \$1.50?

17. Read the following as dollars, cents, and mills:

2765 mills	5639 mills	2793 mills
235 mills	9050 mills	4505 mills

18. Write the above, using the dollar sign.

ROMAN NOTATION.

64. The method of expressing numbers by means of letters is called **Roman Notation**.

It is called Roman Notation because it was originally used by the ancient Romans.

65. In this system, seven capital letters are used to express numbers, viz. :

LETTERS: I, V, X, L, C, D, M.

VALUES: 1, 5, 10, 50, 100, 500, 1000.

By combining these letters according to certain principles, any number can be expressed.

66. PRINCIPLES.—1. *When a letter is repeated, its value is repeated.*

Thus, X represents ten; XX, twenty; XXX, thirty; CC, two hundred; MM, two thousand.

2. *When a letter is placed before another of greater value, its value is to be taken from that of the greater.*

Thus, I represents one, V, five, and X, ten. But IV represents four; IX, nine; XL, forty; XC, ninety.

3. *When a letter is placed after another of greater value, their values are to be united.*

Thus, XV represents fifteen; LXX, seventy; LXXX, eighty; MC, one thousand, one hundred.

4. *When a letter is placed between two letters, each of greater value than itself, its value is to be taken from the sum of the other two.*

Thus, XIV represents fourteen; XIX, nineteen; CIV, one hundred four.

5. *A bar placed over a letter increases its value a thousand fold.*

Thus, V represents five; V, five thousand; LXX represents seventy; LXX, seventy thousand.

TABLE.

I = 1	XIII = 13	LX = 60
II = 2	XIV = 14	LXX = 70
III = 3	XV = 15	XC = 90
IV = 4	XVI = 16	C = 100
V = 5	XIX = 19	CC = 200
VI = 6	XX = 20	CCCC = 400
VII = 7	XXI = 21	CD = 400
VIII = 8	XXIX = 29	D = 500
IX = 9	XXX = 30	DCC = 700
X = 10	XXXIV = 34	M = 1000
XI = 11	XL = 40	MMM = 3000
XII = 12	L = 50	MDCCCXCII = 1892

EXERCISES.

Read the following :

XX	XIX	CDXX	XL	XCIV
LXX	XLIV	XXV	XC	XIX
XCIX	XXXV	LXIV	VIII	MDLIV
XXI	LXIX	CCXXIV	LX	MDCCC
XLIX	XXXVI	CCCLIX	DCXL	MMDC
MDCCCX	CCXXVI	DLXXI	MDXL	DCCL
CXCV	CCXLIV	MMMD	IVDXL	CCXCV

Express the following by Roman Notation :

23	61	84	35	312	517	1010	1900
34	19	59	47	419	493	1800	8000
15	36	62	86	226	499	1492	9000
27	43	97	214	384	278	1607	1721

ADDITION.

67. 1. How many blocks are 5 blocks, 3 blocks, and 7 blocks?

2. How many splints are 3 splints, 2 splints, and 8 splints?

3. How many oranges are 7 oranges, 2 oranges, and 5 oranges?

4. How many are 5 and 6 and 7?

5. How many are 7 and 8 and 6?

6. How many are 5 and 9 and 2 and 6?

7. How many are 3 and 7 and 4 and 8?

8. How many are 4 and 8 and 5 and 9?

9. What have you been doing with the numbers given above?

68. The process of finding a number that is equal to two or more given numbers is called **Addition**.

69. The result obtained by adding is the **Sum**, or **Amount**.

70. The **Sign of Addition** is a small upright cross (+). It is read *plus*, and is placed between the numbers to be added.

Thus, $4 + 2$ is read 4 plus 2, and means that 4 and 2 are to be added.

71. The **Sign of Equality** is two short parallel horizontal lines (=). It is read *equals*, or *is equal to*.

Thus, $3 + 2 = 5$ is read 3 plus 2 equals 5.

WRITTEN EXERCISES.

72. 1. Add 13, 12, 24, 30, and 14.

EXPLANATION. — In adding, the numbers are written in columns, units under units, tens under tens, and it is more convenient to begin with the units' column. Adding, we find there are 13 units. 13 means 1 ten and 3 units; therefore the 3 is written in the units' column, and the 1 ten is added to the tens. Adding the tens, we find that there are 9 tens. Therefore the result is 93.

93. In adding, do not say 4 and 4 are eight and 2 are ten, etc., simply name results, thus: 4, 8, 10, 18.

Find the sum of

2.	3.	4.	5.	6.	7.	8.	9.
----	----	----	----	----	----	----	----

72	73	84	57	\$ 27	\$ 21	\$ 96	\$ 20
----	----	----	----	-------	-------	-------	-------

85	97	17	46	72	53	81	38
----	----	----	----	----	----	----	----

90	16	52	93	18	38	45	47
----	----	----	----	----	----	----	----

36	68	78	16	19	73	78	93
----	----	----	----	----	----	----	----

—	—	—	—	—	—	—	—
---	---	---	---	---	---	---	---

10.	11.	12.	13.	14.	15.	16.	17.
-----	-----	-----	-----	-----	-----	-----	-----

39	54	89	28	59	25	45	68
----	----	----	----	----	----	----	----

29	90	24	77	83	27	17	39
----	----	----	----	----	----	----	----

72	71	74	50	17	48	18	74
----	----	----	----	----	----	----	----

45	29	93	33	26	16	13	56
----	----	----	----	----	----	----	----

13	36	35	86	28	36	14	23
----	----	----	----	----	----	----	----

—	—	—	—	—	—	—	—
---	---	---	---	---	---	---	---

18.	19.	20.	21.	22.	23.	24.	25.
-----	-----	-----	-----	-----	-----	-----	-----

45	62	42	65	43	68	56	42
----	----	----	----	----	----	----	----

58	87	24	89	69	42	49	46
----	----	----	----	----	----	----	----

34	49	85	38	22	25	17	34
----	----	----	----	----	----	----	----

22	36	48	76	85	34	21	58
----	----	----	----	----	----	----	----

67	55	34	45	74	28	36	91
----	----	----	----	----	----	----	----

—	—	—	—	—	—	—	—
---	---	---	---	---	---	---	---

26. Two birds fly in opposite directions, one 19 miles, and the other 27 miles. How far apart are they?
27. Max earned 24 dollars, and John earned 9 dollars more than Max. How many dollars did both earn?
28. Baby is 4 years old; mamma is seven times as old as baby; grandma is twice as old as mamma. What is the sum of their ages?
29. In a store window I saw 38 books, 25 toys, 17 slates, and 20 globes. How many articles did I see?
30. In one class in our school there are 11 boys and 9 girls; in another, 17 boys and 18 girls; in another, 10 boys and 14 girls. How many boys are there in all? How many girls? How many children?
31. A fisherman caught 46 trout, 75 bass, and 13 blue-fish. How many fishes did he catch?
32. Fred weighs 83 pounds, and Nathan weighs 97 pounds. How much do they both weigh?
33. In one book there are 98 pages, in another 89, and in a third 75. How many pages are there in all the books?
34. A certain tract of land was divided into 4 farms; one containing 13 acres, another 37 acres, the third 70 acres, and the fourth 48 acres. How many acres did the original tract contain?
35. The distance from Alburn to Grade is 6 miles, from Grade to Newton 8 miles, from Newton to Arnold 9 miles, and from Arnold to Houston 8 miles. How far is it from Alburn to Houston?
36. Elsie solved 18 examples on Monday, 14 on Tuesday, and 12 on Wednesday. How many examples did she solve in all?
37. If I buy a collar for 25 cents, some gloves for 37 cents, some ribbon for 8 cents, and a paper of needles for 9 cents, what do they all cost?
38. A farmer has in one bin 28 bushels of oats, in another

36, in another 23. How many bushels has he in the three bins?

39. If I sell 8 chickens to one man, 18 to another, and have 24 left, how many had I at first?

40. Mr. Atwater bought a cow for 36 dollars, a sheep for 8 dollars, and a calf for 13 dollars. How much did he pay for all?

41. Susie has 38 cents, and her brother has 17 cents more than she has. How many cents have they both?

42. Morton's geography cost 75 cents, his reader 39 cents, and his slate 18 cents. How much did they all cost?

43. Mr. Barnes has 3 fields. In one he put 36 cows, in another 42 cows, and in the third 27 cows. How many cows has he in the three fields?

44. A fisherman sold 4 fishes for the following sums: the first for 37 cents, the second for 44 cents, the third for 54 cents, the fourth for 48 cents. What did he receive for all?

45. On a Christmas tree I counted 12 dolls, 25 balls, 28 flags, 5 whistles, 3 drums, 25 candy-bags, and 14 books. How many things did I count?

46. Thornton has 68 marbles, Oliver has 10 less than Thornton. How many have both boys together?

47. One stalk of a plant has 43 leaves, a second stalk has 36 leaves, and a third has 18 leaves. How many leaves have they all?

48. A baker made at one time 37 loaves, at another 56, at another 28. How many loaves did he make in all?

49. A milkman sold 72 quarts of milk on Monday, 56 on Tuesday, 85 on Wednesday, and 49 on Thursday. How many quarts did he sell in the four days?

50. A butcher sold 24 pounds of veal, 37 pounds of pork, 38 pounds of chickens, and 49 pounds of beef. How many pounds of meat did he sell?

51. A man has 45 sheep, 28 cows, and 15 calves. How many animals has he?

52. If I spend 8 dollars a week for board, 3 dollars for car fare, and have 14 dollars left of my week's earnings, how much did I earn?

53. Wilfred paid 28 dollars for a desk. He paid 5 dollars to have it polished, and then sold it for 7 dollars more than it cost him. How much did he get for the desk?

54. Commencing with 28, find the sum of 28 and the succeeding numbers less than 36?

55. A cloth merchant sold 5 rolls of cloth, containing respectively 46, 52, 63, 29, and 76 yards. How many yards did he sell?

56. A saddle cost 14 dollars, a wagon 60 dollars more than the saddle, and a horse as much as the saddle and wagon together. How much did the horse cost? How much did all cost?

57. Harold earned 87 cents. If he previously had 36 cents, and then found 7 cents, how many cents had he in all?

58. Howard wrote 38 lines of German on Monday, 27 on Tuesday, 15 on Wednesday, 17 on Thursday, and 26 on Friday. How many lines did he write during the week?

59. A man sold three lots. For one he received \$695, for another \$734, and for the third \$392. How much did he receive for all?

60. A owns a farm of 518 acres, B one of 425 acres, and C one of 392 acres. How many acres do they all own?

61. In a city there were four schools, the first containing 396 pupils, the second 683 pupils, the third 718 pupils, and the fourth as many as the second and third together. How many pupils attended the schools? How many attended the fourth school?

Add the following :

62.	63.	64.	65.	66.	67.	68.
68	54	37	81	39	55	38
45	32	93	39	23	38	43
93	81	86	44	87	37	71
87	36	94	87	65	25	64
94	57	83	39	34	61	83
86	91	27	86	56	39	73
83	87	55	54	83	24	98
<u>37</u>	<u>32</u>	<u>66</u>	<u>37</u>	<u>74</u>	<u>86</u>	<u>72</u>

69.	70.	71.	72.	73.	74.
452	374	418	519	735	875
378	516	372	327	872	912
684	834	384	684	659	386
359	912	596	519	738	596
871	816	387	329	486	837
964	325	426	548	684	542
832	426	385	819	396	816
<u>576</u>	<u>845</u>	<u>713</u>	<u>384</u>	<u>845</u>	<u>347</u>

75.	76.	77.	78.	79.
3785	7351	7638	6835	381
4651	4286	1842	7867	3468
3278	3875	3451	392	193
5964	1894	4265	8437	8200
8137	5768	7169	24	57
8462	3681	8376	6174	93
5738	5437	1452	5762	3956
9124	8796	3814	8600	4184
6783	5106	6842	439	324
3854	7340	6937	3764	47
4265	4165	5843	158	596
<u>7384</u>	<u>9354</u>	<u>7165</u>	<u>99</u>	<u>1938</u>

SUBTRACTION.

73. 1. How many cents has George if he lost 5 cents of the 9 cents which he had ?

2. Ten horses were drawing wagons, all but 5 of which were gray. How many gray horses were there ?

3. Bertha had 13 cents and spent 6 cents. How much had she left ?

4. How many are 10 less 3 ? 15 less 8 ? 17 less 7 ?

5. How many are 16 less 7 ? 19 less 9 ? 13 less 5 ?

6. How many are 12 less 8 ? 14 less 5 ? 15 less 7 ?

7. How many are 13 less 6 ? 17 less 9 ? 18 less 9 ?

8. What have you been doing with the above numbers ?

74. The process of finding the difference between two numbers, or of taking part of a number from it and finding how many are left, is called **Subtraction**.

75. The number from which another is taken is called the **Minuend**.

76. The number to be subtracted is the **Subtrahend**.

77. The number remaining after one number is taken from another is called the **Remainder**.

The remainder is the difference between the *minuend* and the *subtrahend*.

78. The **Sign of Subtraction** is a short horizontal line (−). It is called **Minus**.

Thus, $8 - 3$ is read, 8 minus 3, and means that 3 is to be subtracted from 8.

WRITTEN EXERCISES.

79. 1. From 869 subtract 423.

Minuend 869 EXPLANATION. — For convenience, the less
 Subtrahend 423 number is written under the greater, units under
 — units, tens under tens, etc. Beginning at the right,
 Remainder 446 each order of units in the subtrahend is subtracted
 separately from the same order in the minuend.

Thus, 9 units — 3 units are 6 units, which are written under units.
 6 tens — 2 tens are 4 tens, which are written under the tens.
 8 hundreds — 4 hundreds are 4 hundreds, which are written under
 the hundreds. Hence the remainder is 446.

PROOF. — 446, the remainder, plus 423, the subtrahend, equals 869,
 the minuend. Hence the result is correct.

Copy, subtract, and prove:

2.	3.	4.	5.	6.	7.	8.
888	999	564	899	329	957	864
356	276	213	486	115	516	231
—	—	—	—	—	—	—
9.	10.	11.	12.	13.	14.	15.
435	768	975	543	879	768	599
234	253	432	133	217	642	143
—	—	—	—	—	—	—
16.	17.	18.	19.	20.		
From	876	789	968	999	888	
Take	242	176	723	275	234	
	—	—	—	—	—	—

21. Susie had two bundles of 10 splints each, and 6 more. How many splints had she? She gave her teacher 8 of the splints. How many had she left?

She found the 6 ones were not enough to give her teacher, so she took one bundle of the tens, untied it, and put it with her 6 ones, making 16 ones besides the 1 ten which she had not untied. She took 8 away from the 16 and found she had 8 left, and also the 1 ten, or 18 left in all.

Do just as Susie did.

22. Take 1 bundle of one hundred splints, 4 bundles of 10 splints each, and 5 single splints. How many have you in all? Give your teacher 27 of the splints. Compare what you did with the following solution:

$$\begin{array}{r}
 & \overset{3}{\cancel{1}} \overset{15}{\cancel{5}} \\
 1 & 4 \cancel{5} \\
 & 2 7 \\
 \hline
 & 1 1 8, \text{ Remainder.}
 \end{array}$$

23. Annie had 2 bundles of one hundred splints each. She gave Fred 78. How many had she left?

$$\begin{array}{r}
 & 9 \\
 1 & 10 \cancel{10} \\
 2 & 0 \cancel{0} \\
 & 7 8 \\
 \hline
 & 1 2 2
 \end{array}$$
 Annie took 1 bundle of one hundred, untied it, and found there were 10 tens in it. She left 9 of the tens a moment and untied 1 bundle of 10, finding 10 ones in it. Of these she gave 8 to Fred, and had 2 left. Then taking up the 9 tens, she gave 7 tens to Fred, and had 2 tens left, and besides these the 1 bundle of one hundred. She had, therefore, 122 left.

Find the difference between the following, and supply the numbers omitted in minuend, subtrahend, or remainder:

24.	25.	26.	27.	28.	29.	30.	31.
34	86	92	41	70	83	44	96
17	<u>69</u>	<u>55</u>	<u>18</u>	<u>29</u>	<u>45</u>	<u>28</u>	<u>73</u>
32.	33.	34.	35.	36.	37.	38.	39.
99	72	77	77	91	85	88	65
45	<u>48</u>	<u>43</u>	<u>29</u>	<u>88</u>	<u>17</u>	<u>33</u>	<u>28</u>
40.	41.	42.	43.	44.	45.	46.	47.
871	761	240	166	752	631	382	546
257	<u>347</u>	<u>18</u>	<u>139</u>	<u>239</u>	<u>408</u>	<u>272</u>	<u>338</u>
	48.	49.	50.	51.	52.	53.	54.
Minuend,	300	204	500	607	908	550	600
Subtrahend,	<u>171</u>	<u>78</u>	<u>183</u>	<u>179</u>	<u>127</u>	<u>258</u>	<u>379</u>
Remainder,							

	55.	56.	57.	58.	59.	60.	61.
Minuend,	635	591	624	350	870	980	527
Subtrahend,	469	268	156	123	456	379	318
Remainder,							
	62.	63.	64.	65.	66.	67.	
Minuend,	?	769	?	780	800	?	
Subtrahend,	17	?	238	?	?	?	502
Remainder,	136	144	196	123	191	139	
	68.	69.	70.				
Minuend,	\$ 541	\$ 905	768	children.			
Subtrahend,	\$ 492	\$ 184	599	"			
Remainder,	\$	\$		children.			
	71.	72.	73.				
Minuend,	808 eggs.	967 birds.	842	pints.			
Subtrahend,	527 "	209 "	763	"			
Remainder,	eggs.	birds.		pints.			
	74.	75.	76.				
Minuend,	400 horses.	502 cows.	\$ 550				
Subtrahend,	285 "	237 "	\$ 118				
Remainder,	horses.	cows.	\$				
	77.	78.	79.				
Minuend,	800 bushels.	?	?	days.			
Subtrahend,	269 "	372 "	18	"			
Remainder,	bushels.	apples.	347	days.			
	80.	81.	82.				
Minuend,	75 years.	\$ 772	\$ 324				
Subtrahend,	?	\$ 624	?				
Remainder,	19 years.	\$	\$ 136				

REVIEW EXERCISES.

ADDITION AND SUBTRACTION.

80. 1. From a tank containing 935 gallons of water, 648 gallons were drawn off, after which 247 gallons ran in. How many gallons were there then in the tank?

2. Two trains start from Chicago. One goes east 298 miles, and the other west 247 miles. How far apart are they?

3. A man purchased 8983 bricks, but used only 5362. How many had he left?

4. A man paid \$465 for a carriage. He spent \$57 for repairs on it, and sold it for \$500. Did he gain or lose, and how much?

5. From the sum of 763 and 297 subtract their difference.

6. A horse and cow together cost \$276. If the cow cost \$81, what was the cost of the horse?

7. Two trains start from cities 582 miles apart. If they travel towards each other, one 123 miles and the other 238 miles, how far apart are they?

8. Columbus discovered America in 1492. How many years have passed since then?

9. A merchant bought 30 pieces of cloth, containing in all 979 yards. He afterwards sold all but 143 yards. How many yards did he sell?

10. The cost of building a house was \$3972, and of furnishing it \$1597. What was the entire cost? How much more did it cost to build it than to furnish it?

11. Cotton was first planted in the United States about 1759. How long ago was that?

12. Ralph threw a ball 37 feet, and Jamie threw his 19 feet farther than Ralph. How many feet must Jamie go to pick up his ball and return?

13. A man owned a horse that cost \$852, and a carriage that cost \$437. He was obliged to sell them both for \$1000. How much money did he lose by the sale?
14. My neighbor has in his orchard 264 pear trees, which is 187 less than the number of his apple trees. How many apple trees has he? How many trees has he in all?
15. A farmer having 425 bushels of wheat, sold 127 bushels to one person, and 264 bushels to another. How many bushels had he left?
16. $\$24.03 + \$17.75 + \$19.63 + \$10 + \$18.09 + \$7.99 = ?$
17. $\$10 - (\$2.35 + \$3.78 + \$1) = ?$
18. Mr. Jones earned during July \$172. He paid a bill for \$28 to the grocer, \$19 to the butcher, and \$36 for rent. How much of the money remained?
19. $\$4700 - (\$462 + \$2180) = ?$
 $\$3900 - (\$2614 + \$854.50) = ?$
20. A man gave to his son \$575; to his daughter, \$468; and to his nephew, \$249 less than to his daughter. How much did he give to all?
21. Two men, who are 425 miles apart, travel towards each other, one 36 miles, the other 58 miles a day. How far apart will they be at the end of two days?
22. If the same men travel away from each other, how far apart will they be at the end of two days?
23. In a certain school there are 692 pupils. In the primary grade there are 325; in the intermediate grade, 259; and the remainder are in the grammar grade. How many are in the grammar grade?
24. A man bought a lot of land for \$1860, and built upon it a house costing \$7584. If he sold the property for \$10,000, how much did he make?
25. If I buy at a grocery store sugar for 78 cents, cheese for 39 cents, and nuts for 25 cents, giving in payment a \$2 bill, how much change should I receive?

26. From 946 subtract 486; from the result subtract 284; to this add 398; and from this answer subtract 199.

27. Lena's book has 482 pages. If she reads 168 pages in the morning, and 89 pages in the afternoon, how many more pages has she to read?

28. Bert and Harry had each \$1.40. Bert earned 28¢ more, and Harry spent 72¢ of his money. How much more has Bert than Harry?

29. In a school there were 278 girls and 215 boys. If 29 girls and 34 boys leave, how many pupils will there then be attending the school?

30. Into a bin that holds 825 bushels, I put 272 bushels of corn, and afterwards 346 bushels more. How many more bushels of corn will it hold?

31. If I buy a hat for \$6.75 and give in payment a \$10 bill, what change should I receive?

32. The entire expenses of a journey made by a family were \$391.86. The fare on the railroads was \$192.15, carriage hire was \$37.25, fare on the steamers amounted to \$61.35, and the rest was paid for board. What did the board cost?

33. A merchant failed in business, owing debts to the amount of \$1837.85. His goods on hand were worth \$894.50, and there was due him from persons who had bought goods from him, \$591.75. How much more did he owe than he could pay?

$$34. \$68.95 + \$81.36 - \$29.15 - \$42.36 + \$25.84 = ?$$

$$35. \$76.37 + \$34.93 + \$8.83 - \$16.25 - \$13.24 = ?$$

$$36. \$59.23 + \$18.76 - \$32.17 + \$7.08 - \$41.25 = ?$$

$$37. \$32.94 - \$16.23 + \$13.81 - \$20.06 + \$24.53 = ?$$

$$38. \$68.92 + \$23.75 - \$15.27 - \$6.35 + \$15.18 = ?$$

$$39. \$37.50 - \$18.24 + \$60.08 - \$18.32 - \$16.29 = ?$$

$$40. \$51.23 + \$67. + \$59.38 - \$62.81 - \$4.85 = ?$$

$$41. \$24.69 + \$41.80 - \$23 + \$27.42 - \$8 + \$15 = ?$$

MULTIPLICATION.

81. 1. Henry worked 5 days per week for 5 weeks. How many days did he work in that time?

2. A boy delivered books to 12 houses, leaving 6 books at each house. How many books did he carry altogether?

3. The wages of a laborer were \$12 per week. How much could he earn in 6 weeks?

4. The price of a cord of wood is \$5: How much must be paid for 7 cords?

5. The postman who delivers our mail travels 9 miles per day. How far will he travel in 8 days?

6. A lady bought 11 yards of cambric at 10 cents per yard. How much did it cost her?

7. A good walker can walk 4 miles per hour. How far can he walk in 9 hours at that rate?

8. How many are eight 3's? Nine 5's?

9. How many are seven 7's? Eight 6's?

10. How many are six 8's? Nine 4's?

11. How many are four 6's? Five 4's?

12. What have you been doing with the numbers in the examples given above?

82. A short process of finding the sum of several equal numbers, or the process of taking one number as many times as there are units in another is called **Multiplication**.

83. The number taken or multiplied is called the **Multiplicand**.

84. The number which shows how many times the multiplicand is taken is called the **Multiplier**.

85. The result obtained by multiplying is called the **Product**.

86. The multiplicand and multiplier are called the **Factors** of the product.

87. The **Sign of Multiplication** is an oblique cross, \times . It is read *multiplied by* when the multiplicand precedes it and *times* when the multiplier precedes it.

Thus 6×8 is read 6 multiplied by 8 when 6 is the multiplicand, but it is read 6 times 8 when 6 is the multiplier.

MULTIPLICATION TABLE.

1	2	3	4	5	6	7	8	9	10	11	12
2	4	6	8	10	12	14	16	18	20	22	24
3	6	9	12	15	18	21	24	27	30	33	36
4	8	12	16	20	24	28	32	36	40	44	48
5	10	15	20	25	30	35	40	45	50	55	60
6	12	18	24	30	36	42	48	54	60	66	72
7	14	21	28	35	42	49	56	63	70	77	84
8	16	24	32	40	48	56	64	72	80	88	96
9	18	27	36	45	54	63	72	81	90	99	108
10	20	30	40	50	60	70	80	90	100	110	120
11	22	33	44	55	66	77	88	99	110	121	132
12	24	36	48	60	72	84	96	108	120	132	144

EXPLANATION.—The numbers in the left-hand column may be regarded as the multipliers, and the numbers across the top as the multiplicands. The products will be found in the *horizontal columns* opposite the multipliers.

Thus, 2 ones are 2 ; 2 twos are 4 ; 2 threes are 6 ; 2 fours are 8, etc.

The order may be changed so that the numbers in the upper horizontal line may be regarded as the multipliers, and the numbers on the left as the multiplicands. Each successive multiplier may be used with each multiplicand.

Thus, 1 two is 2 ; 2 twos are 4 ; 3 twos are 6 ; 4 twos are 8, etc.

DRILL EXERCISES.

Tell instantly the following products:

8×4	5×11	12×6	7×5	2×7
5×3	12×7	11×5	2×6	3×4
6×8	9×9	4×9	3×5	2×4
7×2	8×12	5×5	4×2	12×3
9×8	3×3	7×6	5×2	9×4
8×7	4×10	9×7	6×9	10×12
3×9	5×7	10×8	7×11	11×4
4×6	6×5	11×10	7×3	2×3
5×6	11×7	2×11	2×9	8×6
3×7	12×9	9×2	4×4	6×7
4×8	7×8	5×9	6×3	12×8
5×12	9×6	7×4	10×11	10×4
6×11	10×10	9×11	8×11	8×10
7×10	11×11	10×7	7×12	4×7
8×9	12×12	8×8	9×10	10×3
9×5	7×7	4×5	10×6	12×2
5×8	8×5	5×10	11×9	4×12
3×10	2×8	6×12	12×11	3×2
4×3	2×12	7×9	2×5	11×3
3×11	9×3	6×10	3×8	11×2
2×10	3×12	6×6	6×2	11×6
5×4	8×3	12×5	10×5	12×10
10×9	3×6	8×2	11×12	10×2
11×8	6×4	9×12	12×4	4×11

88. When the multiplier is not greater than 12.

ORAL EXERCISES.

1. When oranges cost 4 cents apiece, how much must Mary pay for 6 oranges?
2. If Lulu changes 6 half-dimes into cents, how many cents will she have?
3. Henry went to school every school-day for 8 weeks. How many days was that?
4. A girl bought 9 yards of ribbon for 8 cents a yard? How much did it cost her?
5. An organ-grinder received 7 cents at each of 8 houses. How much did he get in all?
6. A boy sold 9 pencils at 4 cents apiece. How much did he get for them all?
7. In a school-room there were 8 rows of seats, and 8 seats in a row. How many pupils could be seated there?
8. A half-quire of paper contains 12 sheets. How many sheets are there in 6 half-quires?
9. A drawing book cost 12 cents. If each pupil in a class of 9 pupils paid the same price for a drawing book, how much did they all pay?
10. Harry had to pay 9 cents per day for his fare in going to and from school. How much was that per week?
11. Alice planted 9 rows of asters in her flower bed, and each row had 9 plants. How many asters did she set out?
12. Each pupil in a class wrote 7 lines in his writing book. If there were 7 pupils in the class, how many lines did they all write?
13. The cost of a railroad ticket from Ashburn to Graystone is 11 cents. How much will 8 such tickets cost?
14. Some children are required to attend school 6 hours per day. How many hours will they be at school in 7 days?

15. If Mary goes to bed at 8 o'clock at night, and rises at 7 o'clock in the morning, how many hours does she spend in bed in 9 days?
16. Reuben said that he saw 5 flocks of birds, each containing 7 birds. How many birds did he see?
17. A school week is 5 days. How many days are there in 9 school weeks?
18. Sarah made on her slate 9 rows of squares, each row containing 7 squares. How many squares did she make?
19. Henry brought home 5 dozen eggs. How many eggs did he bring?
20. Susie's father worked 9 hours per day for 7 days. How many hours did he work in all?
21. How many are two 2's? Three 2's? Four 2's? Five 2's? etc. Give the table of 2's.
22. Give the table of 3's. Of 4's. Of 5's. Of 6's. Of 7's. Of 8's. Of 9's. Of 10's. Of 11's. Of 12's.
23. Count by 5's from 0 to 100 in this way: 0, 5, 10, 15, 20, 25, 30, etc.
24. Count by 3's from 0 to 36. By 7's from 0 to 84.
25. Count by 6's from 0 to 72. By 9's from 0 to 108.
26. Count by 4's from 0 to 48. By 8's from 0 to 96.
27. Count by 10's from 0 to 110. By 12's from 0 to 144.

WRITTEN EXERCISES.

89. 1. How many are 6 times 397?

Multiplicand, 397 EXPLANATION. — For convenience in multiplying, the multiplier is written under the multiplicand. Multiplier, 6 and we begin at the right to multiply in the following manner:
Product, 2382

6 times 7 units are 42 units. But 42 units are 4 tens and 2 units; therefore, the figure 2 is written in units' place in the product, and the 4 tens are reserved to add to the tens of the product.

6 times 9 tens are 54 tens, plus 4 tens reserved are 58 tens, or 5 hundreds and 8 tens; therefore the figure 8 is written in tens' place in

the product, and the 5 hundreds are reserved to add to the hundreds in the product.

6 times 3 hundreds are 18 hundreds, plus 5 hundreds reserved are 23 hundreds, or 2 thousands and 3 hundreds, which are written in thousands' and hundreds' places in the product. Hence the product is 2382.

Find the products of the following:

2. 5 × 46.	14. 415 × 7.	26. 3245 × 4.
3. 6 × 37.	15. 372 × 8.	27. 2861 × 5.
4. 7 × 28.	16. 846 × 5.	28. 3942 × 3.
5. 6 × 93.	17. 364 × 4.	29. 5687 × 6.
6. 7 × 86.	18. 918 × 3.	30. 3834 × 8.
7. 4 × 98.	19. 325 × 5.	31. 5279 × 4.
8. 3 × 69.	20. 864 × 9.	32. 8168 × 9.
9. 8 × 58.	21. 325 × 4.	33. 3934 × 7.
10. 9 × 43.	22. 816 × 6.	34. 5468 × 5.
11. 6 × 85.	23. 932 × 7.	35. 3173 × 2.
12. 11 × 87.	24. 519 × 11.	36. 4284 × 11.
13. 12 × 92.	25. 837 × 12.	37. 1987 × 12.

38. There are 5280 feet in a mile. How many feet does a boy walk who walks 5 miles?

39. A merchant sold Mrs. Allen 5 pieces of muslin, each containing 39 yards. How many yards did he sell to her?

40. The average wages of 5 clerks were \$37.50 per month. How much did they all earn in a month?

41. When hay sells at \$8.25 per ton, how much must I pay for 8 tons?

42. If a railway train runs 47 miles per hour, how far will it run in 8 hours?

43. Theodore Schuyler & Co. paid each of 5 bookkeepers \$85.25 per month. How much did they pay them all?

44. A laborer found that his expenses were \$23.85 per month. How much will they be in 8 months?

45. A carriage maker sold 8 carriages for \$78.75 apiece. How much did he receive for them all?
46. Mr. A loaned a man some money and received for the use of it, or interest, \$18.35 per year. How much should he receive for the use of it for 5 years?
47. A contractor built 8 houses for which he received \$7325 apiece. How much did he receive for all?
48. The expenses of a large dry-goods store were \$3215.25 per month. How much were they for 8 months?
49. An organ manufacturer sold during 1891 nine church organs, the average price of which was \$7963. How much did his sales amount to that year?
50. The number of papers sold by the publisher of a daily paper averaged 8432 each day. How many were sold in 6 days?
51. A mechanic earned \$87.35 per month for 8 months. How much were his entire earnings in that time?
52. A man sold 9 carriages for \$92.55 apiece. How much did he get for them?
53. The steamship Roduna consumes 125 tons of coal per day. How much will she consume in a voyage lasting 8 days?
54. In one of the cities of the United States, the average number of passengers carried upon the street-cars daily was 38,395. How many were carried in 6 days?
55. The sales of tickets at a railway station for the month of January amounted to \$38,495. If they are the same, on the average, for 7 months, how much will they be?
56. The interest paid for a sum of money was \$384.27 per year. How much interest money was paid in 5 years?
57. The increase in population in a city for the year 1891 was 13,868. At that rate, what will be the increase in 6 years?
58. The average distance sailed by a steamship each voy.

age between New York and Liverpool, England, was 2831 miles. How many miles did she sail in 4 round trips?

59. A gardener sold last year 9765 bunches of asparagus. How many bunches would he sell in 7 years at that rate?

Multiply each of the following numbers by 2, 3, 4, 5, 6, 7, 8, and 9:

60. 5869. 63. 3954. 66. 86,973. 69. 18,572. 72. 48,362.

61. 7296. 64. 3857. 67. 23,547. 70. 42,364. 73. 32,573.

62. 8164. 65. 5679. 68. 65,132. 71. 38,453. 74. 71,685.

90. When the multiplier is expressed by more than one figure.

ORAL EXERCISES.

1. How many are 10 times 6? 10 times 7? 10 times 8? 10 times 5? 10 times 9?

2. How many are 10 times 3 plus 3 times 3 or 13 times 3? 10 times 4 plus 6 times 4 or 16 times 4? 10 times 7 plus 3 times 7 or 13 times 7? 10 times 8 plus 5 times 8 or 15 times 8?

3. How many are 15 times 4? 18 times 3? 17 times 5? 16 times 6? 12 times 8? 14 times 3?

4. How many are 13 times 5? 13 times 6? 13 times 7?

5. What will 15 yards of ribbon cost at 5 cents per yard? At 8 cents per yard?

6. How many days are there in 17 weeks?

7. How many quarts are there in 16 gallons?

8. How many feet are there in 13 yards?

9. How much will 15 yards of calico cost at 6 cents per yard? At 9 cents per yard?

10. How much does a man save in 12 weeks if he saves \$8 per week?

11. There are 9 square feet in a square yard. How many square feet are there in 14 square yards?

12. How many are 10 times 9? 10 times 5? 10 times 7?
13. What figure must be annexed to 9 to multiply it by 10? What to 5? What to 7?
14. How, then, is any number multiplied by 10?
15. How many are 10 sevens or 10 times 7? 100 sevens or 100 times 7? 100 eights or 100 times 8? 100 times 6? 100 times 15? 100 times 35? 100 times 48? 100 times 56? 100 times 84? 100 times 97? 100 times 63?
16. How, then, is any number multiplied by 100?
17. How many are 100 times 9? 1000 times 9? 1000 times 7? 1000 times 15? 1000 times 35? 1000 times 43? 1000 times 54? 1000 times 75?
18. How, then, is any number multiplied by 1000?
19. Since a number is multiplied by 10 by annexing *one* cipher to the number; by 100 by annexing *two* ciphers; by 1000 by annexing *three* ciphers, how may a number be multiplied by 10, 100, 1000, etc., or by 1 with any number of ciphers annexed?
91. *A number is multiplied by 10, 100, 1000, etc., or by 1 with any number of ciphers annexed, by annexing to the multiplicand as many ciphers as there are in the multiplier.*

WRITTEN EXERCISES.

92. What are the products of the following:

- | | | |
|--------------------|-----------------------|---------------------------|
| 1. $45 \times 10.$ | 6. $324 \times 100.$ | 11. $5463 \times 1000.$ |
| 2. $67 \times 10.$ | 7. $416 \times 100.$ | 12. $3784 \times 1000.$ |
| 3. $38 \times 10.$ | 8. $723 \times 100.$ | 13. $5169 \times 10000.$ |
| 4. $46 \times 10.$ | 9. $342 \times 100.$ | 14. $4237 \times 10000.$ |
| 5. $76 \times 10.$ | 10. $518 \times 100.$ | 15. $8193 \times 100000.$ |
16. Multiply $832 \times 3000.$

832

3000 EXPLANATION. — Since 3000 is 1000 times 3, 832 is first multiplied by 3, giving 2496 as a product, and this product is multiplied by 1000 by annexing three ciphers.

2496000

- | | | |
|---------------------|-----------------------|-------------------------|
| 17. $45 \times 20.$ | 27. $346 \times 500.$ | 37. $3946 \times 6000.$ |
| 18. $37 \times 30.$ | 28. $279 \times 300.$ | 38. $5279 \times 4000.$ |
| 19. $68 \times 40.$ | 29. $518 \times 700.$ | 39. $8365 \times 7000.$ |
| 20. $73 \times 80.$ | 30. $264 \times 900.$ | 40. $7226 \times 9000.$ |
| 21. $56 \times 60.$ | 31. $385 \times 400.$ | 41. $5387 \times 8000.$ |
| 22. $90 \times 40.$ | 32. $498 \times 600.$ | 42. $7394 \times 5000.$ |
| 23. $30 \times 50.$ | 33. $341 \times 800.$ | 43. $2869 \times 3000.$ |
| 24. $23 \times 90.$ | 34. $957 \times 200.$ | 44. $9999 \times 2000.$ |
| 25. $84 \times 70.$ | 35. $691 \times 500.$ | 45. $8765 \times 4000.$ |
| 26. $65 \times 60.$ | 36. $444 \times 400.$ | 46. $9182 \times 6000.$ |

47. Multiply $373 \times 235.$

FIRST PROCESS.	373	SECOND PROCESS.	373
	235		235
1 st partial product,	1865		1865
2 ^d partial product,	11190		1119
3 ^d partial product,	74600		746
Entire product,	87655		87655

EXPLANATION OF FIRST PROCESS.—For convenience, the multiplier is written under the multiplicand with units under units, tens under tens, etc.

Since we cannot multiply by 235 at one operation, we multiply by the parts of 235, viz.: 5 units, 3 tens or 30, and 2 hundreds or 200, and add the products.

5 times 373 is 1865, the first partial product; 30 times 373 is 11190, the second partial product; 200 times 373 is 74600, the third partial product. The sum of these partial products is 87655, the entire product.

EXPLANATION OF SECOND PROCESS.—The second process is the one commonly used. It is the same as the first except that the ciphers are omitted from the right of the partial products. The significant figures, however, occupy their proper places.

Thus, in multiplying by 3 tens the product is 1119 tens, and consequently, the right-hand figure of this product is placed under tens and the others in their proper places in the product.

In multiplying by 2 hundreds or any number of hundreds the lowest order of the product is hundreds, consequently the right-hand figure of the product is written under hundreds.

93. From these processes the following general method of multiplying is readily deduced:

1. *The multiplier is written under the multiplicand, units under units, tens under tens, etc.*

2. *Each figure of the multiplicand is multiplied by each significant figure of the multiplier and the right-hand figure of each product is placed under the figure of the multiplier used to obtain it.*

3. *The sum of the partial products will be the entire product.*

When there is a cipher in the multiplier, multiply by the significant figures only, taking care to place the right-hand figure of each partial product under the figure used to obtain it.

48.	49.	50.
5684	78645	59658
609	5008	3700
51156	629160	417606
34104	393225	178974
3461556	393854160	220734600

Multiply:

- | | | |
|----------------|------------------|---------------------|
| 51. 267 by 24. | 66. 3845 by 236. | 81. 5698 by 792. |
| 52. 836 by 27. | 67. 7186 by 314. | 82. 3179 by 865. |
| 53. 918 by 61. | 68. 5327 by 420. | 83. 8169 by 734. |
| 54. 309 by 38. | 69. 6284 by 518. | 84. 5234 by 826. |
| 55. 465 by 27. | 70. 4769 by 344. | 85. 3186 by 839. |
| 56. 732 by 55. | 71. 6873 by 530. | 86. 5279 by 918. |
| 57. 817 by 63. | 72. 2918 by 364. | 87. 3864 by 609. |
| 58. 398 by 84. | 73. 4873 by 273. | 88. 38,675 by 708. |
| 59. 426 by 29. | 74. 9275 by 731. | 89. 42,308 by 692. |
| 60. 918 by 76. | 75. 8462 by 820. | 90. 75,069 by 804. |
| 61. 837 by 84. | 76. 7319 by 394. | 91. 83,427 by 3625. |
| 62. 695 by 32. | 77. 8425 by 972. | 92. 80,069 by 5846. |
| 63. 864 by 69. | 78. 6685 by 873. | 93. 58,325 by 3192. |
| 64. 528 by 87. | 79. 6517 by 762. | 94. 38,164 by 8068. |
| 65. 364 by 44. | 80. 7395 by 469. | 95. 49,234 by 9096. |

96. \$ 48.27 by 584. 101. \$ 543.79 by 3980.
 97. \$ 39.35 by 376. 102. \$ 708.27 by 2900.
 98. \$ 73.86 by 705. 103. \$ 496.83 by 5805.
 99. \$ 85.18 by 835. 104. \$ 385.76 by 3760.
 100. \$ 49.66 by 597. 105. \$ 483.74 by 5736.

106. A manufacturer sold 29 reapers for \$ 96.85 apiece. How much did he get for them ?

107. A man walked 37 miles. How many feet did he walk, since there are 5280 feet in a mile ?

108. A tract of land containing 294 acres was sold for \$ 313 per acre. For how much did it sell ?

109. It required 318 loads of bricks averaging 1387 bricks to build a school building. How many bricks was that ?

110. A railroad employed 213 switchmen, whose wages averaged \$ 78.34 per month. How much did all the switchmen receive per month ?

111. A merchant sold 7 pieces of cloth, each containing 34 yards, for \$ 2.17 per yard. How much did he receive for it ?

112. The Danton Leather Co. purchased 37 lots of hides, each lot containing 384 hides. How many did they purchase ?

113. A cloth dealer sold 54 pieces of cassimere, each containing 48 yards, at \$ 1.87 per yard. How much did he receive for it ?

114. A fire destroyed 345 gross of pencils. Since a gross is 144, how many pencils were destroyed ?

115. In a day there are 1440 minutes. How many minutes are there in a year ?

116. A farm of 95 acres was sold for \$ 37.25 per acre. How much was received for it ?

117. If your pulse beats 4320 times per hour, how many times will it beat in 2 days of 24 hours each ?

118. The cost of building a railroad was \$ 31,755 per mile.

If it was 118 miles long, how much did it cost to construct it?

119. A pump can pump 365,864 gallons per day. How many gallons can it pump in 31 days?

120. The expense of an army in Europe is, on the average, \$195 per man. How much will it cost to keep an army of 32,596 men?

121. If a locomotive costs \$19,325, how much will 23 such locomotives cost?

122. An army used 3285 barrels of flour each day for 49 days. How many barrels did it use altogether?

123. A shoe manufacturer in Massachusetts sold 23,876 pairs of shoes at an average price of \$3.15 a pair. How much did he receive for them?

REVIEW.

ORAL EXERCISES.

94. 1. Henry bought 3 pencils at 3 cents apiece, and 5 postage stamps at 2 cents apiece. How much did he pay for all?

2. The wages of a clerk were \$15 per week. If he paid \$5 for board and \$4 for other expenses, how much did he save in 5 weeks?

3. Nina spent 20 cents for a box of water-color paints, 5 cents for brushes, and had 10 cents left. How much money had she at first?

4. Two men travel in opposite directions from a starting-point, one at the rate of 5 miles per hour, and the other at the rate of 4 miles per hour. How far apart are they at the end of 5 hours?

5. Mary counted the petals on 6 flowers. All but 2 of them had 6 petals each, and the others had 5 each. How many petals had they all?

6. Hannah was at school 6 hours per day for 4 days, and 3 hours on the other day. How many hours was she at school that week?

7. Warren had 5 hens and each hen had 7 chickens. How many chickens were there?

8. A grocer sold 5 pounds of sugar at 5 cents a pound, and received in payment a fifty-cent piece. How much change should he give back.

9. Henry counted a company of soldiers in this way. He found that there were 8 rows of 9 men each and three officers. How many soldiers did he see?

10. Some children found 5 birds' nests containing 5 eggs each, and 4 nests containing 4 eggs each. How many eggs did they find?

11. A farmer sold 5 calves at \$5 each, and 5 sheep at \$4 each. How much did he receive for all?

12. Mary paid 10 cents for a dozen peaches, and 2 cents apiece for a dozen bananas. How much did they all cost her?

13. A boy worked 6 days for 5 dimes per day, but he spent 2 half-dimes for car fare each day. How much did he save?

14. Two brothers worked during the summer vacation of 10 weeks, one for \$6 per week and the other for \$3 per week. How much more did the one earn than the other in that time?

15. The pupils in a school were divided into 8 classes or grades, each containing 9 pupils. If every class contained 5 girls and 4 boys, how many boys were there in the school? How many girls? How many pupils?

16. A merchant employed 5 clerks at \$8 per week, and 2 at \$10 per week. How much did he pay them all per week?

17. The retail price of asparagus was 15 cents a bunch,

and the wholesale price was 12 cents. How much is a retail dealer's profit on 20 bunches which he bought at wholesale?

18. A hatter bought 12 hats at \$4 apiece, and sold them at \$6 apiece. How much did he gain?

19. A clothier sold overcoats at \$12.50 which cost him \$9.50. How much was his gain on 20 coats?

20. After paying \$2 apiece for repairing clocks that cost \$10 apiece, a man sold 12 of them for \$144. Did he gain or lose by the sale, and how much?

WRITTEN EXERCISES.

95. 1. A bookkeeper received \$75 per month for his services, but he spent in a year \$680. How much did he save in a year?

2. A train composed of 23 cars, containing 145 sheep each, arrived at Boston and the sheep were sold at \$5 a head. How much was received for them?

3. A boy had \$350 when he was 15 years old, and he saved \$100 per year until he was 21. How much had he then?

4. A manufacturer employed 118 workmen whose average wages per month were \$68. How much did they all earn per month?

5. A drover bought 83 cattle at \$37.25 per head. He sold 50 of them at \$45 per head, and the rest at \$35.25 per head. How much did he gain?

6. A tea merchant sold 348 pounds of tea at 57 cents per pound and gained \$13.55. How much did it cost him?

7. A and B each began business with \$4000. A gained \$600 the first year and B lost an equal sum. The second year A gained \$318 and B gained \$895. How much was each worth then?

8. A farmer raised 895 bushels of oats. He kept 50 bushels for seed, and enough to keep 10 horses, allowing 15 bushels for each horse, and sold the rest at 35 cents per bushel. How much did he receive for the amount sold?

9. A coal dealer sold 315 tons of coal at \$5.25 per ton. He received in payment a note for \$1000 and the rest in cash. How much cash did he receive?

10. The profits of a merchant were \$5896.75. He paid \$1391 for his household expenses, and 3 times as much for other expenses. What was his net gain?

11. What will be the cost of 485 chests of tea, each containing 55 pounds, at 38 cents a pound?

12. I had a farm of 313 acres. 185 acres yielded me an income of \$6.25 per acre, and the rest \$6 per acre. How much was my entire income from the farm?

13. My agent sold 3 lots of goods. For the first he received \$945, for the second twice as much as for the first, and for the third 3 times as much as for the other two. How much did he receive for all?

14. Three men formed a partnership. A contributed \$690, B twice as much, and C three times as much as A and B. What was their entire capital?

15. A drover bought 132 cattle at \$45 per head, and 67 at \$61 per head. He sold them all at \$50 per head. Did he gain or lose, and how much?

16. A farmer sold his farm at a gain of \$1685. The barns cost him \$1855, his house \$8520, and the land twice as much as the barns and house. How much did he get for the farm?

17. A merchant bought 35 pieces of carpet, each containing 49 yards, at \$1.18 per yard. He paid cash for a part of the cost and a note to settle the account for \$1250. How much cash did he pay?

18. Three men began business at the same time. During the first year A made a profit of \$855, B of 3 times as

much, and C of 4 times as much as A. At the beginning of the second year they formed a partnership in which each man contributed his gain of the previous year. What was the entire capital?

19. The daily sales of a newspaper for 5 days were 16,845 copies. If the papers were sold for 3 cents each, what was the amount received?

20. I know a man whose income is \$85.75 per day. How much does he receive in a year, or 365 days?

21. The annual expense of a school for 5 years was \$3825.25, exclusive of the wages of the teachers. In the school there were 11 teachers whose average annual salary was \$815.85. What was the annual expense of the school?

22. It was discovered in taking a census that a certain town contained 145 houses, that each house, on the average, contained 2 families, and that each family averaged 6 persons. How many people were there in the town?

23. A man bought a farm of 189 acres. He sold 99 acres at \$87 per acre, and thus reduced the cost of the rest to \$58 per acre. How much did the farm cost him?

24. A single poppy has been known to contain 32,000 seeds. How many seeds will 335 such poppies contain?

25. In a codfish 9,334,000 eggs have been found. If from each egg a fish was hatched, how many fishes were hatched from the eggs of 27 such codfish?

26. A man set out on foot to travel 600 miles. He traveled during the first 5 days 45 miles per day; during the next 6 days 38 miles per day. How many miles had he still to travel?

27. A ship left port, sailing at the rate of 9 miles per hour. After sailing for 23 hours she was driven directly back by a storm at the rate of 12 miles per hour for 10 hours. How far was she from the port from which she sailed, at the end of 30 hours?

DIVISION.

- 96.** 1. How many groups of 2 squares can be formed from 6 squares? How many 2's are there in 6?
 2. How many groups of 3 oranges can be formed from 9 oranges? How many 3's are there in 9?
 3. Henry earns 6 cents an hour. In how many hours can he earn 42 cents? How many 6's are there in 42?
 4. Louise arranged 56 spools in rows, placing 7 in each row. How many rows did she form?
 5. Daisy's brother was ill for 49 days. How many weeks was he ill? How many 7's are there in 49?
 6. Anna had 63 chickens which were hatched in broods of 9 chickens each. How many broods were there? How many 9's are there in 63?
 7. A lad rides each day 8 miles in going to and from school. In how many days will he ride 96 miles? How many times is 8 contained in 96?
 8. Mollie arranged 12 books in 4 piles. How many books were there in each pile?
 9. How many apples will each child receive, when 10 apples are divided equally among 5 children?
 10. What have you been doing with the above numbers?
- 97.** The process of finding how many times one number is contained in another, or the process of separating a number into equal parts, is called **Division**.
- 98.** The number to be divided is the **Dividend**.

99. The number by which we divide is the **Divisor**.

100. The result obtained by division is the **Quotient**.

101. The part of the dividend remaining when the division is not exact, is the **Remainder**.

102. The **Sign of Division** is $+$. It is read *divided by*. When placed between two numbers, it shows that the one at the left is to be divided by the one at the right.

Thus, $24 + 6$ is read, 24 divided by 6.

103. Division is also indicated by writing the dividend above the divisor, with a line between them.

Thus, $\frac{24}{6}$ is read, 24 divided by 6.

Another method of indicating division is by writing the divisor at the left of the dividend, with a line between them.

Thus, $6)24$ is read, 24 divided by 6.

DRILL EXERCISES.

104. Give the results instantly:

$9 \times 2 = ?$	$9 \times ? = 18$	$? \times 2 = 18$	$18 + 2 = ?$
$6 \times 7 = ?$	$6 \times ? = 42$	$? \times 7 = 42$	$42 + 7 = ?$
$5 \times 8 = ?$	$5 \times ? = 40$	$? \times 8 = 40$	$40 + 8 = ?$
$7 \times 5 = ?$	$7 \times ? = 35$	$? \times 5 = 35$	$35 + 5 = ?$
$8 \times 3 = ?$	$8 \times ? = 24$	$? \times 3 = 24$	$24 + 3 = ?$
$4 \times 9 = ?$	$4 \times ? = 36$	$? \times 9 = 36$	$36 + 9 = ?$
$6 \times 8 = ?$	$6 \times ? = 48$	$? \times 8 = 48$	$48 + 8 = ?$
$20 + 10$	$21 + 7$	$48 + 12$	$50 + 10$
$8 + 4$	$48 + 4$	$24 + 4$	$27 + 9$
$20 + 2$	$42 + 6$	$16 + 2$	$66 + 6$
$96 + 8$	$44 + 11$	$24 + 6$	$60 + 12$
$6 + 3$	$28 + 4$	$30 + 5$	$63 + 7$
$22 + 11$	$36 + 4$	$66 + 11$	$48 + 8$
$12 + 2$	$70 + 10$	$8 + 2$	$49 + 7$
$20 + 4$	$63 + 9$	$81 + 9$	$35 + 7$
			$24 + 3$

24 + 12	88 + 8	35 + 5	90 + 10	45 + 9
18 + 3	55 + 11	80 + 10	48 + 6	108 + 9
60 + 6	84 + 7	10 + 2	110 + 11	121 + 11
15 + 3	16 + 4	77 + 11	80 + 8	108 + 12
56 + 7	14 + 7	6 + 2	99 + 9	21 + 3
90 + 9	4 + 2	36 + 9	40 + 8	132 + 12
70 + 7	16 + 8	84 + 12	50 + 5	120 + 10
72 + 8	72 + 12	18 + 2	27 + 3	110 + 10
20 + 5	30 + 3	54 + 9	100 + 10	120 + 12
12 + 6	18 + 6	12 + 3	24 + 8	132 + 11
33 + 11	10 + 5	12 + 4	9 + 3	144 + 12
40 + 4	15 + 5	33 + 3	36 + 6	42 + 7
25 + 5	30 + 10	32 + 4	77 + 7	60 + 10
30 + 6	24 + 2	40 + 10	99 + 11	56 + 8
18 + 9	72 + 6	64 + 8	28 + 7	36 + 3
54 + 6	36 + 12	55 + 5	72 + 9	14 + 2

105. When the divisor is not greater than 12.

ORAL EXERCISES.

- When coal is selling at \$5 per ton, how many tons can be bought for \$45?
- How many rows of trees are there in an orchard containing 60 trees arranged in rows containing 6 trees each?
- If a railroad ticket to Chicago costs \$7, how many such tickets can be bought for \$35?
- A girl paid 96 cents for muslin at 8 cents per yard. How many yards did she buy?
- How many dictionaries, at \$8 apiece, can be bought for \$64?
- How long will it take a boy to save \$48 if he saves \$4 per month?
- Howard paid \$9 for a suit of clothes. How many such suits can be bought for \$54?
- How many gallons of molasses are there in $4\frac{3}{4}$ quarts?

9. A peck measure holds 8 quarts of grain. How many pecks are there in 72 quarts?
10. If 8 nails are required to fasten a horseshoe, how many horseshoes can be fastened with 80 nails?
11. How many pairs of boots, at \$6 a pair, can be bought for \$48?
12. A farmer had 63 bales of hay. If he drew it away by drawing 7 bales at a load, how many loads did it make?
13. A grocer put 96 quarts of vinegar into two-gallon jugs. How many jugs did he use?
14. A stick measured 72 inches in length. How many feet long was it?
15. Henry found that he had added 66 figures, and that there were 6 figures in each column. How many columns did he add?
16. Mary knew that there were 72 desks in her school-room, and that there were 8 desks in a row, but she could not tell how many rows there were. Can you tell?
17. A grocer had 76 eggs. How many dozen did he have, and how many over?
18. A lad saw 48 birds sitting upon 6 telegraph wires, and he noticed that the same number sat upon each wire. How many were there in each group?
19. Lottie had saved 47 cents which she wished to divide equally, as far as possible, among four charitable societies. How much did she give each, and how much had she left?
20. How many times can a four-quart measure be filled from 49 quarts? How much will be left?
21. A postman found that he walked 24 miles per day in delivering mail. If he made 4 trips per day, how many miles did he walk each trip?
22. Susie spends $\frac{1}{3}$ of the year with her aunt and the rest of the year at home. How many months does she spend with her aunt?

WRITTEN EXERCISES.

106. 1. Divide 1702 by 3.

Divisor. Dividend.

3)1702

567 — 1, Rem.

Quotient.

EXPLANATION.—For convenience, the divisor is written at the left of the dividend, and the quotient below it or above it.

We begin at the left to divide. Thus, 3 is not contained in 1 thousand any thousand times, therefore the quotient cannot contain units of any order higher than hundreds. Hence, we find how many times 3 is contained in all the hundreds of the dividend.

1 thousand plus 7 hundreds are 17 hundreds ; 3 is contained in 17 hundreds 5 hundred times and 2 hundreds remainder. The 5 is written in hundreds' place in the quotient, and the remainder, 2 hundreds, is united with the tens.

2 hundreds plus 0 tens are 20 tens ; 3 is contained in 20 tens 6 times, and a remainder of 2 tens. The 6 is written in tens' place in the quotient, and the remainder, 2 tens, is united with the units.

2 tens plus 2 units are 22 units ; 3 is contained in 22 units 7 times, and a remainder of 1. The 7 is written in units' place in the quotient, and the remainder at a little distance to the right ; or it may be written as a part of the quotient by writing the divisor under it.

Hence, the quotient is 567 and 1 remainder, or $567\frac{1}{3}$.

PROOF.—567, the quotient, multiplied by 3, the divisor, plus 1, the remainder, is equal to 1702, the dividend. Hence, the work is correct.

107. When examples in division are solved without writing the products or remainders, the process is called **Short Division**.

Short division is not generally employed when the divisor exceeds 12.

Divide by short division, and prove :

2.	3.	4.	5.
<u>4)3624</u>	<u>6)65814</u>	<u>7)59273</u>	<u>8)614653</u>
906	10969	84674	768314
6. 4872 + 4.	11. 3936 + 4.	16. \$1125 + 9.	
7. 6830 + 5.	12. 3234 + 3.	17. \$1477 + 7.	
8. 2970 + 5.	13. 5256 + 6.	18. \$4986 + 9.	
9. 2976 + 6.	14. 1362 + 3.	19. \$6765 + 5.	
10. 4635 + 3.	15. 1328 + 8.	20. \$3836 + 7.	

21.	9872 + 6.	28.	\$ 9185 + 4.	35.	<u>1882</u> 3
22.	4567 + 5.	29.	\$ 84364 + 7.	36.	<u>1422</u> 6
23.	\$ 89.32 + 6.	30.	\$ 38857 + 8.	37.	<u>2787</u> 7
24.	\$ 81.74 + 9.	31.	\$ 56878 + 6.	38.	<u>1886</u> 4
25.	\$ 83424 + 8.	32.	\$ 91341 + 9.	39.	<u>52688</u> 4
26.	\$ 73265 + 5.	33.	\$ 72144 + 6.	40.	<u>58691</u> 8
27.	\$ 58624 + 7.	34.	\$ 31486 + 11.	41.	<u>52688</u> 12

42. How many coats, at \$5 each, can be bought for \$2250?

43. If a man spends \$7 per week, how long will it take him to spend \$1477?

44. The salaries of 5 men amounted to \$3765. What was the average salary?

45. If a ship sails 8 miles per hour, how long will it take her to sail 1248 miles?

46. A stage traveled 6 miles per hour. How many hours would it require to travel 2352 miles?

47. At a church collection, each person contributed, on the average, 7 cents. If the entire sum collected was 4823 cents, how many persons contributed?

48. How many yards of cloth, at 9 cents per yard, can be bought for 29,439 cents?

49. How many cloaks, at \$5 each, can be bought for \$32,615?

50. In measuring the depth of the sea, 6 feet are called a fathom. How many fathoms deep is the sea when it measures 3516 feet?

Divide each of the following numbers by 3, 4, 5, 6, 7, 8, and 9.

51.	1950.	55.	5616.	59.	17286.	63.	277223.
52.	1476.	56.	6909.	60.	11615.	64.	548337.
53.	4360.	57.	2616.	61.	44854.	65.	389648.
54.	2943.	58.	1884.	62.	18809.	66.	261950.

108. When the divisor is expressed by more than one figure.**ORAL EXERCISES.**

1. At 10 cents a yard, how many yards of cambric can be bought for 40 cents?
2. There are 25 cents in a quarter of a dollar. To how many quarter dollars are 50 cents equal?
3. If I pay \$ 60 for 20 lambs, how much do I pay for each?
4. A real estate dealer divided 30 acres into 10 equal lots. How many acres were there in each?
5. If a newsboy sells 40 papers per day, in how many days will he sell 120 papers?
6. If I sell books at 30 cents each, how many must I sell to receive 90 cents?
7. How many 10's are there in 60? In 90? In 120? In 250? In 350? In 480?
8. Since in 60 there are 6 tens; in 90, 9 tens; in 120, 12 tens; in 250, 25 tens, how may any number be divided by 10?
9. How many 100's are there in 600? In 900? In 1200? In 3600? In 7500? In 8400?
10. Since in 600 there are 6 hundreds; in 900, 9 hundreds; in 1200, 12 hundreds; in 3600, 36 hundreds, how may any number be divided by 100?
11. How many 1000's are there in 8000? In 25000?
12. Since in 8000 there are 8 thousands, in 25000 there are 25 thousands, how may a number be conveniently divided by 1000?

How may a number be divided by 10, 100, 1000, or by 1 with any number of ciphers annexed?

109. A number may be divided by 10, 100, 1000, or by 1 with any number of ciphers annexed, by cutting off from the right of the dividend as many figures as there are ciphers on the right of the divisor.

WRITTEN EXERCISES.

110. Divide:

1. $1|0)3968|0$
3968

2. $1|00)382|00$
382

3. $1|00)468|57$
468

4. 3720 by 10. 9. 57386 by 100. 14. 52830 by 1000.
 5. 4860 by 10. 10. 42570 by 100. 15. 32685 by 1000.
 6. 3879 by 10. 11. 29830 by 100. 16. 97237 by 1000.
 7. 4265 by 10. 12. 34206 by 100. 17. 54862 by 1000.
 8. 3842 by 10. 13. 51309 by 100. 18. 69154 by 1000.

19. Divide 38476 by 500.

$5|00)384|76$ EXPLANATION.—The number is divided by 100 by cutting off the two right-hand figures of the dividend. Dividing 384 by 5 is, therefore, dividing 38,400 by 500. The quotient is, therefore, 76 and 4 hundreds remainder. This remainder, added to the remainder obtained by dividing by 100, gives the entire remainder, 476; or the quotient is $76\frac{476}{500}$.

Divide:

20. 6835 by 20. 24. 98746 by 300. 28. 83426 by 500.
 21. 7291 by 30. 25. 38295 by 400. 29. 31284 by 800.
 22. 4863 by 60. 26. 41283 by 700. 30. 67395 by 700.
 23. 3925 by 80. 27. 61579 by 900. 31. 81293 by 600.

32. Divide 15989 by 37.

Divisor. Dividend. Quotient.

$$37)15989(432\frac{5}{37}$$

$$\begin{array}{r} 148 \\ 118 \\ \hline 111 \\ 79 \\ \hline 74 \\ \hline 5 \end{array}$$

EXPLANATION.—When the divisor is larger than 12 the steps in the solution are usually written. Thus, 37 is contained in 159 *four* times and a remainder of 11. The 4 is written in the quotient and the next figure of the dividend annexed to the remainder, making 118.

37 is contained in 118 *three* times and a remainder of 7. The 3 is written in the quotient and the next figure of the dividend

annexed to the remainder, making 79.

37 is contained in 79 *two* times and a remainder of 5. Therefore, the quotient is $432\frac{5}{37}$.

When the steps in the solution of an example are written, the process is called **Long Division**.

111. From the solution of the example given, the following are seen to be the successive steps taken in solving an example in Long Division :

Write the divisor at the left of the dividend, with a curved line between them.

Find how many times the divisor is contained in the fewest figures on the left hand of the dividend that will contain it, and write the quotient at the right of the dividend or above it. Multiply the divisor by this quotient, and place the product under the figures divided. Subtract the result from the partial dividend used, and to the remainder annex the next figure of the dividend.

Divide as before until all the figures of the dividend have been annexed to the remainder.

If any partial dividend will not contain the divisor, write a cipher in the quotient, then annex the next figure of the dividend, and proceed as before.

If there is a remainder after the last division, write it after the quotient, or with the divisor under it as part of the quotient.

PROOF. — *Multiply the divisor by the quotient, and to the product add the remainder, if any. If the work is correct, the result will equal the dividend.*

1. The quotient figure may generally be readily obtained by finding how many times the *first figure* of the divisor is contained in the *first figures* of the dividend that will contain it, *making allowance* for the addition to the product from the *second figure* of the divisor.

2. If the product of the divisor by the quotient figure is greater than the partial dividend from which it is to be subtracted, the quotient figure is *too large*.

3. Each remainder must be less than the divisor; otherwise the quotient figure is *too small*.

4. When there is no remainder, the divisor is said to be *exact*.

33.

83) 51875(625

$$\begin{array}{r}
 498 \\
 \hline
 207 \\
 166 \\
 \hline
 415 \\
 415 \\
 \hline
 \end{array}$$

34.

74) 42683(576 $\frac{1}{2}$

$$\begin{array}{r}
 370 \\
 \hline
 568 \\
 518 \\
 \hline
 503 \\
 444 \\
 \hline
 59
 \end{array}$$

35.

52) 45297(871 $\frac{5}{8}$

$$\begin{array}{r}
 416 \\
 \hline
 369 \\
 364 \\
 \hline
 57 \\
 52 \\
 \hline
 5
 \end{array}$$

Find the quotients of:

36. $4202 + 11.$

40. $2310 + 42.$

44. $2747 + 41.$

37. $2556 + 12.$

41. $3796 + 52.$

45. $5084 + 62.$

38. $1386 + 21.$

42. $6840 + 15.$

46. $3367 + 91.$

39. $1488 + 31.$

43. $1728 + 12.$

47. $3024 + 84.$

Divide each of the following numbers by 21, 32, 43, 54, 65:

48. 38425.

53. 64136.

58. 35794.

63. 703821.

49. 65484.

54. 99874.

59. 13570.

64. 492532.

50. 94653.

55. 20562.

60. 88992.

65. 669046.

51. 45677.

56. 71117.

61. 46823.

66. 749638.

52. 84732.

57. 40505.

62. 95895.

67. 838327.

Divide each of the following numbers by 37, 28, 19, 46, 35:

68. 37905.

72. 29823.

76. 19198.

80. 469832.

69. 35911.

73. 50847.

77. 34440.

81. 269724.

70. 98634.

74. 72556.

78. 66984.

82. 123461.

71. 48208.

75. 96054.

79. 85745.

83. 777738.

Divide:

84. 50854 by 94.

86. 559108 by 202.

85. 56394 by 78.

87. 1547250 by 2134.

DRILL TABLE.

<i>A.</i>	<i>B.</i>	<i>C.</i>	<i>D.</i>	<i>E.</i>	<i>F.</i>	<i>G.</i>
1.	4	7	42	294	882	4410
2.	5	4	36	144	576	3456
3.	6	3	24	72	360	3240
4.	7	8	40	320	960	7680
5.	5	7	56	392	1568	15680
6.	2	8	72	576	1152	4808
7.	3	3	27	81	729	5103
8.	8	7	28	196	1960	9800
9.	4	8	64	512	1536	3072
10.	3	5	40	200	1800	5400
11.	6	6	36	180	7200	28800
12.	7	9	27	243	1701	8505
13.	9	8	48	384	768	2304
14.	8	6	36	216	864	3456
15.	8	8	56	448	2240	4480
16.	3	7	63	441	3528	7056
17.	9	7	49	343	3773	15092
18.	4	6	42	252	1008	5040
19.	8	9	54	486	2430	7290
20.	4	5	40	200	1000	9000
21.	9	9	36	324	972	8748
22.	6	8	24	192	768	3840
23.	5	9	81	729	1458	3916
24.	3	8	72	576	2804	9216
25.	5	6	36	324	1296	6480

EXERCISES UPON THE TABLE.

ORAL EXERCISES.

112. 1. $A \times B = ?$ 2. $C + B = ?$ 3. $C - B = ?$

WRITTEN EXERCISES.

- | | | |
|------------------------|-------------------------------|----------------------|
| 4. $C + D + E = ?$ | 9. $D \times C = ?$ | 14. $E + B = ?$ |
| 5. $C + D + E + F = ?$ | 10. $C \times 10 = ?$ | 15. $E + C = ?$ |
| 6. $E - D = ?$ | 11. $D \times 400 = ?$ | 16. $F + C = ?$ |
| 7. $F - E = ?$ | 12. $D \times 203 = ?$ | 17. $F + D = ?$ |
| 8. $D \times B = ?$ | 13. $D + B = ?$ | 18. $100000 - F = ?$ |
| 19. $G + D = ?$ | 21. $G - (D + E + F) = ?$ | |
| 20. $F \times D = ?$ | 22. $A \times B \times C = ?$ | |

REVIEW EXERCISES.

ORAL EXERCISES.

- 113.** 1. A bookseller bought 15 books for \$30 and sold them at a gain of \$15. How much apiece did he get for them?
2. A miller sold 8 barrels of flour at \$5 a barrel and bought with the proceeds cloth at \$4 per yard. How many yards did he purchase?
3. The cost of 10 tons of coal was \$50, but I sold it at a gain of \$20. How much did I receive per ton?
4. A bookseller bought 12 dictionaries at \$8 apiece and sold them at \$10 apiece. How much did he gain?
5. A steam yacht ran up a river for 8 hours at 10 miles per hour, and it came down in 5 hours. What was its rate per hour coming down?
6. The cost of 5 tons of coal at \$5 per ton was \$7 less than the cost of 8 cords of wood. What did the wood cost per cord?
7. The expense of sending delegates to a convention was \$65. If the entire expense had been \$1 less, the expense to each member would have been \$2. How many members were there?
8. A man owed a debt amounting to \$84. If he paid \$60 of the sum in 5 months, in how long a time will he pay it all at that rate?
9. A farmer sold a grocer 12 pounds of butter at 20¢ a pound and took in exchange brooms at 24 cents apiece. How many brooms did he get?
10. Two boys set out from the same place and traveled in the same direction, Arthur traveling 5 miles per hour and Bennie 3 miles per hour. How soon will Arthur be 18 miles ahead of Bennie?
11. How far has each traveled when Arthur is 24 miles ahead?

12. Suppose they travel in opposite directions at those rates, how far apart will they be in 8 hours?
13. In how many days can 4 men do as much work as 8 men can do in 6 days?
14. Howard's father gave him an acre of land to till so that he might earn his own pocket money. He sold the products of the land for \$38, but he paid $\$5\frac{1}{2}$ for seeds to plant and $\$2\frac{1}{2}$ for help. How much money had he left?
15. A boy bought 6 lead-pencils for 24¢ and exchanged them for some erasers at 3¢ each. How many erasers did he get?
16. A man bought a carriage for \$120. After spending \$40 in repairing and painting it, he sold it for \$150. Did he gain or lose, and how much?
17. A newsboy bought 20 papers for 30 cents and sold them at a gain of 30 cents. For how much apiece did he sell them?
18. Henry bought 6 car-tickets for 25 cents. How many could he buy for 50 cents?
19. A clerk saved \$20 a month. In how many months could he save enough to buy a library worth \$240?
20. A jeweler bought 12 watches for \$72 and sold them at a gain of \$3 each. At what price did he sell each of them?
21. The wages of a machinist were \$21 per week, and his expenses were \$10. In how many weeks can he pay a debt of \$75 and deposit \$46 in a savings bank?

WRITTEN EXERCISES.

- 114.** 1. If a man paid \$7564 for 124 horses, how much did each cost?
2. At \$9 a yard, how many yards of velvet can I buy for \$207?
3. How many weeks are there in 1736 days?

4. If 12 horses cost \$1560, what will one horse cost ?
What will 5 horses cost at the same rate ?
5. How many dozen eggs are there in 1728 eggs ?
6. Divide \$800.25 equally among 15 men.
7. In an orchard there are 25 rows of trees. If there are 3200 trees in all, how many trees are there in a row ?
8. How many times 26 bushels are there in 22,568 bushels ?
9. How many acres of land will \$22,575 buy, at \$75 an acre ?
10. In 64 loads there were 143,360 pounds. How many pounds were there in a load ?
11. How many days are there in 3600 hours ? In as many minutes ?
12. How many hours are there in 4320 minutes ? How many days ?
13. What number, multiplied by 23, will give 36,087 ?
14. Product, 62,397 ; multiplier, 27 ; find the multiplicand.
15. If the product of two factors is 21,015, and one factor is 45, what is the other factor ?
16. If you weigh 1476 ounces, how many pounds (of 16 ounces) do you weigh ? How many pounds does your sister weigh, her weight being 133 ounces less than yours ?
17. If the receipts of a railroad for one year (52 weeks) are \$2,683,928, how much are they per week ?
18. The combined wealth of 405 men is \$1,247,400. How much would each have if the money were equally divided among them ?
19. How many schooners, each carrying 8700 bushels of wheat, will be required to carry 843,900 bushels ?
20. Suppose that two cans of equal size together hold 376 pints. How many gallons are there in each ?
21. How many poor families may be supplied from 37

barrels of flour, allowing 28 pounds to each, a barrel of flour weighing 196 pounds ?

22. How many thousands make one million ?
23. If a railroad company pays \$ 1750 for constructing a car, how many cars can be made for \$ 75,250 ?
24. The average daily receipts of a ferry-boat are \$ 325. In how many days will the receipts amount to \$ 76,050 ?
25. The stock of a railroad company is \$ 2,565,000. If it is divided into shares of \$ 500 each, how many shares are there ?
26. Mount Everest, in Asia, is said to be 29,100 feet high. Since there are 5280 feet in a mile, how many miles high is it ?
27. In what number is 224 contained 28 times ?
28. Mr. Smith earns \$ 28 while Mr. Allen earns \$ 15. How much will Mr. Smith earn while Mr. Allen earns \$ 120 ?
29. How many times will a cart-wheel, 15 feet in circumference, revolve in going a mile, or 5280 feet ?
30. How many posts and how many rails will be required for a fence 312 feet long, if the posts are set 12 feet apart, and the fence is 4 rails high ?
31. From a farm containing 1000 acres, there were sold at one time 274 acres; at another time, 238 acres. How many acres remained unsold ?
32. Divide \$ 432 between Mr. A. and Mr. B., so that Mr. B. will have \$ 16 more than Mr. A.
33. At an election, the sum of the votes received by two opposing candidates was 6742. If the successful candidate received 328 more votes than his opponent, how many votes did each receive ?
34. Into a tank were poured 76 barrels of kerosene, of 43 gallons each. If 2489 gallons are drawn from the tank, how many gallons are left ?

35. If I buy 8 boxes of starch, each box containing 24 pounds, at 3 cents a pound, and offer in payment a \$20 bill, what change shall I receive?

36. Elsie and Lottie have together \$5.70, and Lottie has 40 cents more than Elsie. How much money has each?

37. If 14 men can do a piece of work in 56 days, how long will it take 28 men to do half the work?

38. I bought two horses. The first cost \$312, and the second \$40 less than the first. How much did the two horses cost?

39. If 19 tons of coal run an engine 703 miles, how far will 17 tons run it?

40. A farmer exchanged 4 cows, worth \$79 each, for a span of horses. What were the horses worth apiece?

41. A horse cost \$235, a carriage \$219, and a h顷k 3 times as much as both. What did all three cost?

42. A merchant bought 42 bales of cloth, each bale containing 36 pieces, and each piece 38 yards, at 9 cents a yard. He sold the whole at 11 cents a yard. How much did he gain?

43. If I take 764 bushels from 2000 bushels twice, how much will remain?

44. A drover with \$2144 bought as many horses as possible for \$165 each, and spent the remainder for sheep at \$4 a head. How many of each did he buy?

45. How many times 945 will make 23,625?

46. Twenty men built a school-house in 56 days. In how many days could 70 men have built it?

47. A man bought 163 barrels of flour at \$9 a barrel; 15 barrels were spoiled, and the remainder he sold at \$11 a barrel. Did he gain or lose, and how much?

48. A school uses 36 crayons a day. Since a gross is 12 dozen, how many gross will it use in 40 weeks, provided there is no school on Saturday?

49. If it takes 5 yards of cloth to make a pair of shirts, what will 24 pairs cost at 15 cents per yard for the cloth, 45 cents apiece for bosoms, wristbands, and buttons, and 75 cents apiece for making?

50. A clerk has a salary of \$42 a month, and his expenses are \$27 a month. How many years will it take him to lay up \$900?

51. Find the cost of 267 yards of cotton at 13 cents a yard, and twice as much at 12 cents a yard.

52. There are in a certain school 37 pupils 14 years old; 76 pupils 12 years old; 114, 11 years old; 149, 10 years old; and 168, 9 years old. What is their average age?

53. A man paid \$375 for a piano, \$13 for freight and cartage, and \$2 for tuning it. He then rented it for 7 quarters at \$15 a quarter, and afterwards sold it for \$325. Did he gain or lose, and how much?

54. Two men leave the same place and travel in opposite directions, one at the rate of 30 miles a day, and the other at the rate of 37 miles per day. How far will they be apart at the end of 14 days?

55. If the men traveled in the same direction, how far apart would they be in the same time?

56. A, B, and C have together \$198. A and C have \$145. A and B have \$99. How much has each?

57. Howard, Roy, and Jamie went fishing, and together caught 77 fish. Of these, Howard and Jamie together caught 58. Howard's and Roy's, when put together, numbered 54. How many did each catch?

58. A farmer sold 18 calves at the rate of 3 for \$33. How much did he get for them?

59. If 117 tons of coal are worth \$702, how much are 235 tons worth?

60. If 12 barrels of flour are worth \$132, what will 36 barrels cost?

61. If a farmer receives \$2275 for 13 horses, for how much would 27 horses sell at the same rate?

62. A's farm contains 227 acres; B's lacks 12 acres of being twice as much; C's lacks 42 acres of being 3 times as much as B's; and D's contains as much as A's and C's. How many acres are there in each farm?

63. A farmer wished to obtain \$260. He sold 72 bushels of wheat at \$1.50 per bushel, and enough apples at \$2 per barrel to obtain the sum required. How many barrels of apples did he sell?

64. If a bookkeeper receives \$1400 per year for his services, and his expenses are \$840, in what time can he save enough to buy 32 acres of land at \$140 per acre?

65. A man starting out on a journey took \$200. He paid for railroad fare \$67; for berth in sleeping car, 4 days, \$2 a day; for hotel bills, 15 days, \$3 a day; and for other expenses, \$25. How much money had he left?

66. The greater of two numbers is 4056, and their difference is 3650. What is the less number?

67. The divisor is 534, and the quotient 43. What is the dividend?

68. The subtrahend is 34,203, and the remainder is 8706. What is the minuend?

115. The parenthesis, (), shows that the numbers included within it are to be subjected to the same operation.

Thus, $(15 - 3) \times 3$ means that $15 - 3$, or 12, is to be multiplied by 3; but $15 - 3 \times 3$ means that 15 is to be diminished by 3×3 , or 9.

$(7 + 2) \times 5$ means that the sum of 7 and 2 is to be multiplied by 5; but $7 + 2 \times 5$ means that 7 is to be increased by 2×5 .

$(7 + 5) + 6$, or $\frac{7 + 5}{6}$, means that the sum of 7 and 5 is to be divided by 6.

$[(9 + 3) \times 2 - 8] + 7$ means that the sum of 9 and 3 is to be multiplied by 2, the product diminished by 8, and the remainder divided by 7.

$(7 \times 8) - (5 \times 5)$ means that the product of 7 and 8 is to be diminished by the product of 5 and 5.

116. The Vinculum, —, or brackets [], may be used instead of the parenthesis.

Thus, $\overline{5 - 3} \times 3$ or $[\overline{5 - 3}] \times 3$ may be used instead of $(5 - 3) \times 3$.

117. In finding the value of expressions in which the parenthesis, vinculum, or brackets are used,

First, perform the operations on the numbers that are written within parenthesis and brackets, or under the vinculum, as indicated by the signs. Next, multiply and divide, as indicated by the signs \times and \div . Finally, add and subtract, as indicated by the signs $+$ and $-$.

Find the values of the following:

1. $(4 + 7) \times 5.$
2. $4 + 7 \times 5.$
3. $(8 - 3) \times 2.$
4. $8 - 3 \times 2.$
5. $4 + 16 + 4.$
6. $(4 + 16) \div 4.$
7. $(3 + 4) \times (8 - 5).$
8. $\frac{7 + 5}{3} + \frac{7 - 3}{4}.$
9. $2 \times 12 + 2 \times 3.$
10. $2 \times 12 + (2 \times 3).$
11. $18 - \frac{(3 \times 4) - (2 \times 3)}{3}.$
12. $[(5 + 6) \times 4 - 5 \times 8] \times 9.$
13. $(3 \times 4) \times 5 - (9 + 7) \div 8.$
14. $2 + 12 \div 4 + (8 + 8 + 4) + 2.$
15. $(312 + 36) - (381 - 215) - 65.$
16. $(214 - 81) - (115 - 18 + 6) + 10.$
17. $(413 - 200) - (118 - \overline{24 - 4 + 6}) + 3.$
18. $(171 - 86) - (\overline{3 \times 4 + 27}) + 10.$
19. $(3 \times 4) \times 9 - (4 + 12) + 4 + 5.$
20. $(5 + 2 + 6) \times 4 - (7 + 8 - 3) + 6 + 10.$
21. $\overline{(36 - 7) \times 5 + (102 + 6) \div 9}.$
22. $\overline{(99 - 3) + 8 - (86 + 10) + 12 + (3 + 6) + 3}.$

FACTORING.

- 118.** 1. What is the product when 3 and 2 are multiplied together? What are 3 and 2 of their product? (Art. 86.)
2. What factors will produce 9?
3. What numbers when multiplied together will produce 12? What are 3 and 4, or 6 and 2 of their product?
4. What are the factors of 20? Of 36? Of 15?
5. What are the factors of 27? Of 25? Of 32?
6. What are the factors of 21? Of 33? Of 50?
7. What are the factors of 63? What else may 7 be called of 63 besides a factor?
8. If 9 is one of two factors of 18, what is the other factor? If 3 is one of the factors? If 6 is one of the factors?
9. What numbers will exactly divide 18? 25? 36?
10. Give the exact divisors of 42; 96; 35; 50; 27; 72.
11. Give the factors of 36; 40; 48; 70; 80.
12. Give the exact divisors of 44; 56; 64; 84; 96.
13. Name the exact divisors of 49; 88; 63; 24; 27.
14. What numbers between 0 and 10 cannot be exactly divided by any number except themselves and 1? What numbers between 10 and 20? Between 20 and 30?
15. What numbers between 0 and 10 can be exactly divided by other numbers besides themselves and 1? Between 10 and 20? Between 20 and 30?
16. Select from the following the numbers that have no exact divisors except themselves and 1: 35, 42, 63, 56, 61, 47, 49, 81, 37, 26, 18, 45.

17. Select from the following the numbers that have exact divisors besides themselves and 1: 24, 36, 41, 39, 27, 45, 33, 37, 50, 44, 60, 71, 72.

119. A number that expresses whole units is called an **Integer**.

Thus, 5, 27, 35 are integers, or integral numbers.

120. The integers which, upon being multiplied together will produce the number, are called **Factors** of the number.

Thus, 5 and 3 are the factors of 15.

121. An integer which will divide a number without a remainder is called an **Exact Divisor** of the number.

Thus, 2, 3, 6, and 9 are exact divisors of 18. They are also *factors* of 18.

122. A number that has no exact divisors except itself and 1 is called a **Prime Number**.

Thus, 1, 3, 5, 7 are prime numbers.

123. A number that has exact divisors besides itself and 1 is called a **Composite Number**.

Thus, 24, 36, 40, 100 are composite numbers.

124. Factors that are prime numbers are **Prime Factors**.

Thus, 7 and 5 are the prime factors of 35.

125. A number that is exactly divisible by 2 is called an **Even Number**.

Thus, 8, 12, 20, 24 are even numbers.

126. A number that is not exactly divisible by 2 is called an **Odd Number**.

Thus, 15, 21, 35, 43 are odd numbers.

127. The process of separating a number into its factors is called **Factoring**.

TESTS OF DIVISIBILITY.

- 128.** 1. Make a list of numbers from 1 to 126, which have 2 for one or more of their factors or divisors. Notice what the right-hand, or units' figure, of each is.
2. Make a list of numbers from 1 to 100 which have 5 for one or more of their factors or divisors. Notice what the units' figure of each is.
3. Make a list of numbers from 1 to 100 which have 3 for one or more of their factors or divisors. Divide the sum of the digits of each of these numbers by 3, and notice the remainder, if any.
4. Make a list of numbers from 1 to 200 which have 9 for one or more of their factors or divisors. Divide the sum of the digits of each by 9, and notice the remainder, if any.

- 129.** It is apparent, therefore, that:

A number is divisible by

2 if the units' figure is 2, 4, 6, 8, or 0.

5 if the units' figure is 0 or 5.

3 if the sum of its digits is divisible by 3.

9 if the sum of its digits is divisible by 9.

- 130.** Tell by inspection which of the numbers, on page 129, columns *C* and *D*, are divisible by 2; by 5; by 9. Which in column *E* are divisible by 2; by 3; by 5.

WRITTEN EXERCISES.

- 131.** 1. What are the prime factors of 336?

2	336
2	168
2	84
2	42
3	21
	7

EXPLANATION.—Since every factor of a number is a divisor of it, we may find the *prime* factors of 336 by dividing by the *exact* divisors that are *prime* numbers. Dividing by 2, we find the factors of 336 to be 2 and 168. But 168 has a factor 2, and, since a factor of a factor of

336 = $2 \times 2 \times 2 \times 2 \times 3 \times 7$ a number is a factor of the number itself, we continue the process, and obtain the prime factors of 336.

What are the prime factors

- | | | |
|-------------|--------------|--------------|
| 2. Of 168? | 12. Of 625? | 22. Of 1280? |
| 3. Of 224? | 13. Of 912? | 23. Of 2625? |
| 4. Of 144? | 14. Of 832? | 24. Of 3204? |
| 5. Of 198? | 15. Of 990? | 25. Of 1155? |
| 6. Of 216? | 16. Of 1008? | 26. Of 2500? |
| 7. Of 484? | 17. Of 435? | 27. Of 1485? |
| 8. Of 576? | 18. Of 222? | 28. Of 1296? |
| 9. Of 432? | 19. Of 672? | 29. Of 2016? |
| 10. Of 672? | 20. Of 775? | 30. Of 2875? |
| 11. Of 396? | 21. Of 289? | 31. Of 2890? |

CANCELLATION.

ORAL EXERCISES.

- 132.** 1. How many times is 2 times 5 contained in 4 times 5? 2 times 3 in 4 times 3? 2 times any number in 4 times that number?
2. How many times is 4 times 7 contained in 8 times 7? 4 times 35 in 8 times 35? 4 times a certain number in 8 times the same number?
3. How many times is 6×12 contained in 18×12 ? 5×23 in 15×23 ? 7×47 in 21×47 ?
4. What is the quotient of $(24 \times 17) + (12 \times 17)$? Of $(63 \times 24) + (9 \times 24)$? Of $(48 \times 61) + (24 \times 61)$? Of (36×19) divided by (18×19) ?
5. In determining the quotient, what numbers may be omitted from both dividend and divisor?

- 133.** From the solution of the examples given, it is evident that:

Rejecting equal factors from both dividend and divisor does not alter the quotient.

WRITTEN EXERCISES.

- 134.** 1. Divide $4 \times 6 \times 8 \times 10$ by $2 \times 3 \times 4 \times 15$.

$$\begin{array}{r} 2 \ 2 \ 2 \ 2 \\ 4 \times 6 \times 8 \times 10 \\ \hline 2 \times 3 \times 4 \times 15 \\ 1 \ 1 \ 1 \ 3 \end{array} = \frac{16}{3} = 5\frac{1}{3}.$$

EXPLANATION. — The dividend is written above the divisor, with a line between them as in division.

Since the factors 2, 3, and 4 are factors of 4, 6, and 8 in the dividend, we reject them from both dividend and divisor, leaving the factors 2, 2, and 2 in the dividend. Since the factor 5 is common to 10 in the dividend and to 15 in the divisor, it is also rejected, leaving 2 in the dividend, and 3 in the divisor.

The product of the uncanceled factors of the dividend is 16, and of the divisor 8. Hence, the quotient is $1\frac{1}{3}$, or $16 \div 8$, or $5\frac{1}{3}$.

2. Divide $7 \times 4 \times 11 \times 8$ by $22 \times 4 \times 4 \times 21$.

$$\begin{array}{r} 1 \\ 1 \ 1 \ 1 \ 2 \\ 7 \times 4 \times 11 \times 8 \\ \hline 22 \times 4 \times 4 \times 21 \\ 2 \ 1 \ 1 \ 3 \\ 1 \end{array} = \frac{1}{3}.$$

EXPLANATION. — In this example, the common factor 7 is rejected from 7 and 21, leaving 1 in the dividend and 3 in the divisor.

Next, the common factor 4 is removed from 4 in the dividend and 4 in the divisor. Then the common factor 4 is removed from 8 in the dividend and 4 in the divisor, leaving 2 in the dividend and 1 in the divisor. Then the common factor 11 is rejected from 11 in the dividend and from 22 in the divisor, leaving 1 in the dividend and 2 in the divisor. Then the common factor 2 is removed from the dividend and the divisor.

The product of $1 \times 1 \times 1 \times 1$ in the dividend is 1. The product of $1 \times 1 \times 1 \times 3$ in the divisor is 3. Hence, the quotient is $1 + 3$, or $\frac{1}{3}$.

1. When the factor is contained once in any number, the 1 may be omitted; but it is better to write it until the process is clearly understood.

2. When all the factors of both divisor and dividend are canceled, the quotient is 1, for the dividend will then exactly contain the divisor once.

Find the value of:

3. $\frac{4 \times 5 \times 7 \times 9}{2 \times 2 \times 6 \times 7 \times 3}.$

4. $\frac{8 \times 9 \times 12 \times 16}{4 \times 3 \times 5 \times 6 \times 20}.$

$$\begin{array}{ll}
 5. \frac{2 \times 3 \times 8 \times 12 \times 24}{6 \times 4 \times 36 \times 4} & 7. \frac{15 \times 20 \times 25 \times 27}{10 \times 15 \times 18 \times 25} \\
 6. \frac{18 \times 24 \times 32 \times 36}{9 \times 48 \times 4 \times 18} & 8. \frac{25 \times 30 \times 40 \times 35}{20 \times 15 \times 18 \times 7}
 \end{array}$$

Divide, using cancellation :

9. $40 \times 48 \times 54 \times 60$ by $30 \times 24 \times 72 \times 3$.
10. $30 \times 49 \times 64 \times 25$ by $15 \times 35 \times 24 \times 10$.
11. $12 \times 60 \times 36 \times 70$ by $28 \times 48 \times 6 \times 5$.
12. $32 \times 36 \times 33 \times 45$ by $24 \times 30 \times 44 \times 9$.
13. $30 \times 36 \times 24 \times 42$ by $45 \times 27 \times 8 \times 28$.
14. $27 \times 32 \times 45 \times 36$ by $18 \times 24 \times 9 \times 6$.
15. $45 \times 28 \times 36 \times 56$ by $49 \times 4 \times 18 \times 18$.
16. $27 \times 28 \times 35 \times 49$ by $18 \times 21 \times 40 \times 28$.
17. $45 \times 60 \times 63 \times 28$ by $27 \times 20 \times 25 \times 21$.
18. $55 \times 36 \times 27 \times 42$ by $12 \times 25 \times 35 \times 33$.
19. $36 \times 64 \times 25 \times 40$ by $32 \times 50 \times 18 \times 10$.
20. $56 \times 18 \times 32 \times 49$ by $16 \times 36 \times 42 \times 28$.
21. $32 \times 45 \times 72 \times 15$ by $64 \times 18 \times 36 \times 30$.
22. $64 \times 15 \times 63 \times 24$ by $30 \times 21 \times 56 \times 12$.
23. $44 \times 20 \times 50 \times 36$ by $55 \times 10 \times 25 \times 18$.
24. $40 \times 48 \times 21 \times 18$ by $56 \times 36 \times 35 \times 24$.
25. $40 \times 27 \times 32 \times 3$ by $5 \times 8 \times 12 \times 8 \times 27$.
26. A lady bought 9 yards of ribbon at 56 cents per yard, but exchanged it for other ribbon at 32 cents per yard. How many yards did she get?
27. At the rate of \$129 for 27 acres of land, what will 180 acres cost?
28. At what price per yard will 5 bales of cloth, containing 12 pieces of 42 yards each, pay for 50 rolls of carpeting, of 75 yards each, at \$2.10 per yard?
29. A merchant purchased 13 firkins of butter, each containing 39 pounds, at 32 cents a pound, and paid for it with

4 patterns of silk of 13 yards each. How much was the silk worth per yard?

30. A grocer exchanged 27 chests of tea, each containing 48 pounds, worth 40 cents a pound, for 18 pieces of dress goods, each containing 60 yards. What was the price per yard of the dress goods?

31. A stationer had 28 packages of cards, each containing 24 cards, which he exchanged for 21 packages of note paper, each containing 24 sheets, worth 2 cents per sheet. What were the cards worth apiece?

32. Four farms, each containing 80 acres, worth \$72 per acre, were exchanged for 5 farms, each containing 96 acres. What was the value per acre of the farms received in exchange?

33. A farmer exchanged 8 bushels of wheat, at \$2 per bushel, for 3 pieces of muslin, worth 10 cents per yard. How many yards were there in each piece?

$$34. \frac{5 \times 8 \times 7 \times 12 \times 16 \times 24 \times 10 \times 9}{3 \times 16 \times 18 \times 4 \times 20 \times 6 \times 5 \times 8} = ?$$

$$35. \frac{9 \times 8 \times 11 \times 16 \times 15 \times 35 \times 42 \times 18}{6 \times 21 \times 7 \times 5 \times 25 \times 32 \times 4 \times 9} = ?$$

$$36. \frac{12 \times 9 \times 30 \times 25 \times 16 \times 36 \times 48}{18 \times 6 \times 15 \times 35 \times 72 \times 24 \times 9} = ?$$

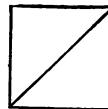
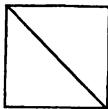
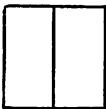
$$37. \frac{42 \times 35 \times 18 \times 24 \times 60 \times 64 \times 72}{36 \times 32 \times 12 \times 25 \times 30 \times 21 \times 5} = ?$$

$$38. \frac{44 \times 20 \times 36 \times 54 \times 65 \times 27 \times 48}{24 \times 40 \times 27 \times 72 \times 13 \times 45 \times 72} = ?$$

$$39. \frac{45 \times 60 \times 63 \times 28 \times 35 \times 24 \times 50}{25 \times 21 \times 56 \times 21 \times 40 \times 48 \times 36} = ?$$

$$40. \frac{36 \times 48 \times 21 \times 35 \times 42 \times 96 \times 56}{28 \times 84 \times 42 \times 24 \times 70 \times 14 \times 9} = ?$$

FRACTIONS.



135. 1. Into how many parts are these squares divided?
2. How do the parts compare as to size?
3. What is each part called?
4. What is a half of anything?
5. How many halves are equal to a whole one?
6. How many halves are there in 2 apples? In 3 pears? In 5 peaches? In 1 orange and a half? In 2 lemons and a half? In 4 and a half plums?
7. Two half melons are equal to how many whole melons? 4 half melons? 6 half melons? 3 half melons are equal to how many whole melons, and how many half melons besides? 5 half melons?
8. Draw a line. Divide it into halves. Then divide each half into two equal parts.
9. Into how many parts is the whole line divided?
10. What is each part called?
11. What do you call 2 of the parts? 3 of the parts? How many thirds are equal to a whole thing?
12. How many fourths are there in any whole thing?
13. How many fourths are there in 2 circles? In 3 squares? In 1 line and 1 fourth of a line? In 2 lines and 3 fourths of a line?

14. How many fourths of an apple are there in half of an apple?
15. What part of a half is a fourth?
16. Which is the greater, 1 half of a yard or 2 fourths of a yard? 1 pineapple or 3 fourths of a pineapple?
17. How many whole apples are equal to 8 fourths of an apple? To 12 fourths? How many whole apples and how many fourths besides are equal to 5 fourths? To 7 fourths? To 9 fourths? To 10 fourths?
18. Draw a circle. Divide it into fourths. Then divide each fourth into 2 equal parts.
19. Into how many equal parts is the circle divided?
20. What is each part called?
21. What do you call 2 of the parts? 3 of the parts? 5 of the parts? 6 of the parts? 7 of the parts?
22. How many eighths are there in a pie? In 2 pies? In 2 pies and 1 eighth of a pie? In 3 pies and 3 eighths of a pie? In 1 fourth of a pie? In 3 fourths of a pie? In 1 pie and 3 fourths of a pie? In half a pie? In 2 and a half pies?
23. How many whole sticks of candy are equal to 8 eighths? 16 eighths? How many whole sticks of candy and how many eighths besides are equal to 10 eighths? To 15 eighths? To 17 eighths? To 20 eighths?
— — — — —
24. What is each part of this line called?
25. What do you call 2 of these parts?
26. How many thirds are there in the whole of anything?
27. How many thirds are there in 3 lines? In 5 lines? In 2 lines and 1 third of a line? In 4 and 2 thirds lines?
28. If I have 6 thirds of orange, to how many whole oranges is that equal? 9 thirds? 12 thirds?
— — — — —
29. How is this line divided?
30. What is each part called? What

do you call 2 parts? What other name will express the same part of the line that 2 sixths expresses?

31. What do you call 3 parts of this line? What other name will express the part of the line that 3 sixths expresses?

32. What do you call 4 parts of this line? What other name will express the part of the line that 4 sixths expresses?

33. How many sixths are there in 2 sheets of paper? 3 sheets? 4 sheets? 5 sheets?

34. Which is the greater, 1 third of a pie, or 1 sixth of a pie?

35. What part of a third is a sixth?

36. How many sixths are there in two thirds? In 1 peach and 1 third of a peach? In 2 and a half peaches? In 3 and 5 sixths peaches?

37. How many units and how many sixths besides are there in 7 sixths? 9 sixths? 12 sixths? 14 sixths?

38. Draw a line. Divide it into sixths. Now divide each sixth into 2 equal parts.

39. Into how many equal parts is the line divided?

40. What is each part called?

41. How many twelfths are there in the whole of anything?

42. How many twelfths of a line are equal to a sixth? To a third? To a half? To 2 thirds? To 5 sixths? To 1 and 5 sixths? To 2 and a half? To 2 and 2 thirds?

43. How many units and how many twelfths besides are equal to 15 twelfths? To 18 twelfths? To 20 twelfths?

44. What part of a sixth is a twelfth? What other name will express the part of the line that 2 twelfths expresses? 4 twelfths? 6 twelfths? 8 twelfths? 10 twelfths?

45. Draw three parallel lines of just the same length.

46. Divide the first one into halves, the second one into fourths, and the third one into eighths.
47. Draw three other parallel lines of just the same length as the first group.
48. Divide the first one into thirds, the second, into sixths, and the third into twelfths.
49. Which is the greater. 1 half or 1 third ?
50. Which is the greater, 1 half or 2 thirds ?
51. When you divide the halves into 2 equal parts, what parts of the whole do you have ?
52. When you divide the thirds into 2 equal parts, what parts of the whole do you have ?
53. One third is equal to how many sixths ?
54. Two thirds are equal to how many sixths ?
55. Three thirds are equal to how many sixths ?
56. What is 1 half of 1 half ? 1 half of 1 third ?
57. Which is the greater, 1 third or 1 fourth ?
58. Which is the greater, 1 fourth or 1 sixth ?
59. Which is the greater, 1 third or 2 sixths ?
60. What parts of the whole do you get by dividing the fourths into 2 equal parts ?
61. One fourth is equal to how many eighths ?
62. Two fourths are equal to how many eighths ?
63. Three fourths are equal to how many eighths ?
64. Which is the greater, 1 sixth or 1 eighth ?
65. How many sixths are equal to a half ?
66. How many eighths are equal to a half ?
67. Which is the greater, 3 sixths or 3 eighths ?
68. What parts of the whole do you get by dividing the sixths into 2 equal parts ?
69. One sixth is equal to how many twelfths ?
70. Two sixths are equal to how many twelfths ?
71. Three sixths are equal to how many twelfths ? Four sixths ? 5 sixths ? 6 sixths ?

72. Which is the greater, 1 twelfth or 1 eighth ?
73. How many twelfths are equal to a half ?
74. How many eighths are equal to a half ?
75. Which is the greater, 6 twelfths or 6 eighths ?
76. When anything is divided into 5 equal parts, what is each part called ?
77. What do you call 3 of the parts ?
78. How many fifths are there in the whole of anything ?
79. Draw a line and divide it into fifths. Divide each fifth into 2 equal parts. Into how many equal parts is the line divided ?
80. What is each part called ?
81. How many tenths are there in the whole of anything ?
82. How many tenths are equal to a fifth ?
83. Draw a line and divide it into thirds. Divide each third into 3 equal parts. Into how many equal parts have you divided the line ? What is each part called ?
84. How many ninths are equal to a third of the line ?

136. One or more of the equal parts of anything is a Fraction.

137. Two numbers written one above the other with a line between them are used to express a fraction.

138. The number which shows into how many equal parts the unit has been divided is called the **Denominator**.

It is written below the line.

Thus, in the fraction $\frac{4}{5}$, 5 is the denominator. It shows that the unit has been divided into five equal parts.

139. The number which shows how many parts form the fraction is called the **Numerator**.

It is written above the line.

Thus, in the fraction $\frac{4}{5}$, 3 is the numerator. It shows that the fraction contains 3 of the 5 equal parts into which the unit was divided.

140. The numerator and denominator are called the Terms of a Fraction.

141. A fraction in which the numerator is less than the denominator is called a Proper Fraction.

Thus, $\frac{2}{3}$, $\frac{3}{5}$, $\frac{1}{2}$ are proper fractions.

The value of a proper fraction is, therefore, less than 1.

142. A fraction in which the numerator equals or exceeds the denominator is called an Improper Fraction.

Thus, $\frac{5}{2}$, $\frac{7}{3}$, $\frac{11}{5}$ are improper fractions.

The value of an improper fraction is, therefore, 1 or more than 1.

143. A number expressed by an integer and a fraction is a Mixed Number.

Thus, $6\frac{1}{4}$, $8\frac{2}{3}$, $6\frac{11}{12}$ are mixed numbers.

144. A fraction may be regarded as expressing *unexecuted division*.

Thus, $\frac{15}{4}$ is equal to $15 \div 4$; $\frac{24}{6}$ is equal to $24 \div 6$.

145. 1. Read and interpret the expression $\frac{5}{13}$.

EXPLANATION. — $\frac{5}{13}$ represents 5 of the 13 equal parts into which the unit has been divided. It also represents one thirteenth of 5, and 5 divided by 13. It is read *five thirteenths*.

Read and interpret the following expressions :

- | | | | | |
|--------------------|--------------------|----------------------|-----------------------|----------------------|
| 2. $\frac{8}{5}$. | 5. $\frac{7}{4}$. | 8. $\frac{7}{10}$. | 11. $\frac{11}{5}$. | 14. $\frac{11}{8}$. |
| 3. $\frac{5}{6}$. | 6. $\frac{8}{3}$. | 9. $\frac{5}{12}$. | 12. $\frac{12}{7}$. | 15. $\frac{8}{5}$. |
| 4. $\frac{8}{7}$. | 7. $\frac{4}{7}$. | 10. $\frac{8}{15}$. | 13. $\frac{13}{14}$. | 16. $\frac{16}{9}$. |

Express by figures :

17. Five sevenths. Ten fourteenths. Seven twelfths.
18. Eight ninths. Nine tenths. Five thirteenths.
19. Nineteen twenty-fifths. Thirty-eight forty-fifths.
20. Thirty thirty-ninths. Forty-one sixty-eighths.
21. Forty-six eightyths. Twenty-seven forty-fifths.
22. Five two-hundred-sixths. Seven three-hundred-ninths.
23. One eighth of 17. One thirty-fifth of 59.
24. Fifteen divided by 17. Twenty-four divided by 83.

REDUCTION OF FRACTIONS.

146. To reduce integers or mixed numbers to fractions.

1. A man received \$2 for a day's work. If he was paid in half dollars, how many did he get?

2. How many quarter dollars are equal to \$2?
3. How many sixths are there in 2 pies? Eighths?
4. In three things how many halves are there? Fourths? Eighths? Thirds? Sixths? Twelfths? Fifths? Tenthths?
5. How many fourths are there in 2? In $2\frac{1}{4}$? In $2\frac{3}{4}$?
6. How many eighths are there in 1? In $3\frac{1}{2}$? In $3\frac{3}{4}$?
7. How many sixths are there in 3? In $1\frac{1}{2}$? In $2\frac{1}{2}$?
8. How many fifths are there in 2? In $2\frac{1}{2}$? In $3\frac{1}{2}$?
9. How many sevenths are there in 1? In $1\frac{1}{2}$? In $4\frac{1}{2}$?

147. The process of changing the form of fractions without changing their value is called Reduction of Fractions.

WRITTEN EXERCISES.

148. 1. Reduce $12\frac{3}{8}$ to eighths.

$12 = \frac{96}{8}$. EXPLANATION. — Since in 1 there are 8 eighths, in 12 there are 12 times 8 eighths, or $\frac{96}{8}$; and in $12 + \frac{3}{8} = \frac{99}{8}$. there are $\frac{96}{8} + \frac{3}{8}$, or $\frac{99}{8}$. Hence, $12\frac{3}{8}$ is equal to $\frac{99}{8}$.

149. From the above solution it is seen that:

In changing a mixed number to a fraction, the integer is multiplied by the given denominator, to this product the numerator of the fractional part is added, and the result is written over the given denominator.

Reduce the following to improper fractions:

- | | | | |
|----------------------|------------------------|-------------------------|-------------------------|
| 2. $6\frac{5}{11}$. | 7. $13\frac{3}{7}$. | 12. $36\frac{5}{14}$. | 17. $53\frac{9}{10}$. |
| 3. $9\frac{3}{5}$. | 8. $18\frac{5}{9}$. | 13. $44\frac{7}{12}$. | 18. $65\frac{17}{20}$. |
| 4. $7\frac{4}{9}$. | 9. $21\frac{6}{11}$. | 14. $35\frac{8}{23}$. | 19. $74\frac{11}{17}$. |
| 5. $8\frac{7}{12}$. | 10. $32\frac{7}{10}$. | 15. $37\frac{18}{25}$. | 20. $87\frac{36}{41}$. |
| 6. $9\frac{3}{10}$. | 11. $18\frac{6}{81}$. | 16. $24\frac{16}{33}$. | 21. $98\frac{11}{14}$. |

150. To reduce improper fractions to integers or mixed numbers.

1. To how many oranges are 4 half oranges equal? 6 half oranges? 8 half oranges? 5 half oranges?
2. To how many dollars are 4 quarter dollars equal? 8 quarter dollars? 16 quarter dollars? 5 quarter dollars?
3. In 10 half bushels how many bushels are there? In 12 half bushels? In 20 half bushels?
4. How many units are there in 3 thirds? In 6 thirds? In 12 thirds? In 15 thirds? In 7 thirds?
5. How many units are there in $\frac{8}{3}$? $\frac{9}{3}$? $\frac{12}{3}$? $\frac{14}{3}$? $\frac{18}{3}$?
6. How many yards are there in $\frac{8}{3}$ yards? $\frac{12}{3}$ yards? $\frac{12}{6}$ yards? $\frac{18}{3}$ yards? $\frac{18}{6}$ yards? $\frac{24}{3}$ yards?
7. How many units are there in $\frac{8}{3}$? $\frac{9}{3}$? $\frac{10}{3}$? $\frac{12}{3}$? $\frac{14}{3}$? $\frac{17}{6}$? $\frac{18}{3}$? $\frac{24}{3}$? $\frac{24}{6}$? $\frac{30}{6}$? $\frac{36}{6}$?

WRITTEN EXERCISES.

151. 1. Reduce $17\frac{5}{7}$ to a mixed number.

$17\frac{5}{7} = 125 + 7 = 17\frac{5}{7}$. EXPLANATION.—Since 7 sevenths are equal to 1 unit, 125 sevenths are equal to as many units as 7 sevenths are contained times in 125 sevenths, or 17 $\frac{5}{7}$ units. Therefore, $17\frac{5}{7} = 17\frac{5}{7}$.

152. From the above solution, it is seen that:

In reducing an improper fraction to a mixed number the numerator is divided by the denominator.

Reduce to integers or mixed numbers.

- | | | | |
|----------------------|----------------------|----------------------|----------------------|
| 2. $1\frac{5}{6}$. | 7. $2\frac{14}{3}$. | 12. $6\frac{1}{4}$. | 17. $9\frac{2}{3}$. |
| 3. $2\frac{5}{6}$. | 8. $2\frac{6}{2}$. | 13. $8\frac{2}{7}$. | 18. $5\frac{7}{8}$. |
| 4. $3\frac{2}{3}$. | 9. $4\frac{3}{4}$. | 14. $2\frac{3}{5}$. | 19. $2\frac{2}{5}$. |
| 5. $1\frac{6}{7}$. | 10. $5\frac{1}{7}$. | 15. $2\frac{3}{4}$. | 20. $4\frac{2}{3}$. |
| 6. $5\frac{2}{10}$. | 11. $4\frac{9}{4}$. | 16. $8\frac{1}{5}$. | 21. $5\frac{1}{3}$. |

153. To reduce fractions to smaller or larger terms.

1. In $\frac{1}{4}$ of a yard how many fourths of a yard are there? How many sixths? Eighths? Tenthths?

2. In $\frac{1}{3}$ of an orange how many sixths are there? How many ninths? Twelfths?

3. How are the terms of the fraction $\frac{1}{4}$ obtained from those of $\frac{1}{2}$? $\frac{2}{3}$ from $\frac{1}{3}$?

4. Since the terms of the fraction $\frac{1}{4}$ are obtained from those of $\frac{1}{2}$ by multiplying them by 2, how may the terms of $\frac{1}{3}$ be obtained from those of $\frac{1}{2}$?

5. Since the terms of the fraction $\frac{2}{3}$ are obtained from those of $\frac{1}{3}$ by multiplying them by 3, how may the terms of $\frac{1}{4}$ be obtained from those of $\frac{1}{3}$?

154. From these examples, it is seen that:

The value of a fraction is not changed by multiplying or dividing both terms by the same number.

6. Change to twelfths: $\frac{1}{2}$; $\frac{1}{3}$; $\frac{2}{3}$; $\frac{1}{4}$; $\frac{2}{4}$; $\frac{1}{6}$; $\frac{5}{6}$.

7. Change to twentieths: $\frac{1}{2}$; $\frac{1}{4}$; $\frac{2}{4}$; $\frac{1}{5}$; $\frac{2}{5}$; $\frac{1}{10}$; $\frac{7}{10}$.

8. Change $\frac{2}{3}$ to halves; $\frac{2}{3}$ to fourths; $\frac{1}{2}$ to thirds.

9. Change to thirds: $\frac{2}{3}$; $\frac{1}{3}$; $\frac{2}{3}$; $\frac{1}{3}$; $\frac{1}{3}$; $\frac{2}{3}$; $\frac{1}{3}$; $\frac{2}{3}$.

10. Change to fifths: $\frac{1}{5}$; $\frac{1}{5}$; $\frac{1}{5}$; $\frac{1}{5}$; $\frac{1}{5}$; $\frac{2}{5}$; $\frac{3}{5}$; $\frac{4}{5}$.

155. A number that is an exact divisor of two or more numbers is a **Common Divisor** of those numbers.

156. A fraction is expressed in its **Smallest or Lowest Terms** when its numerator and denominator have no common divisor.

WRITTEN EXERCISES.

157. 1. Reduce $\frac{48}{64}$ to its smallest terms.

$\frac{8}{8})48 = \frac{6}{8}$ EXPLANATION.—Since the fraction is to be reduced to its smallest terms, we divide the terms of the fraction by 8, and the terms of the resulting fraction by 2. The terms $\frac{2}{2})6 = \frac{3}{4}$ of the fraction $\frac{1}{4}$ have no common divisor; therefore $\frac{2}{2})8 = \frac{4}{4}$, when reduced to its smallest terms, is equal to $\frac{1}{2}$.

158. It is evident, therefore, that:

In reducing a fraction to its smallest terms, the numerator and denominator are divided by any common divisor, and this process is continued until the terms have no common divisor.

Reduce to their smallest terms:

2. $\frac{18}{36}$.	8. $\frac{44}{48}$.	14. $\frac{44}{100}$.	20. $\frac{54}{144}$.
3. $\frac{15}{45}$.	9. $\frac{44}{48}$.	15. $\frac{17}{170}$.	21. $\frac{180}{420}$.
4. $\frac{12}{42}$.	10. $\frac{88}{80}$.	16. $\frac{84}{280}$.	22. $\frac{150}{375}$.
5. $\frac{20}{80}$.	11. $\frac{16}{48}$.	17. $\frac{160}{280}$.	23. $\frac{200}{480}$.
6. $\frac{12}{42}$.	12. $\frac{16}{48}$.	18. $\frac{60}{140}$.	24. $\frac{150}{375}$.
7. $\frac{24}{64}$.	13. $\frac{48}{48}$.	19. $\frac{120}{120}$.	25. $\frac{375}{1050}$.

159. To reduce dissimilar to similar fractions.

1. If I bought $\frac{1}{2}$ of a pound of one kind of tea, and $\frac{1}{3}$ of a pound of another kind, how many sixths of a pound of each kind did I buy?
2. Mr. Roe planted $\frac{1}{2}$ of an acre with potatoes, and $\frac{1}{3}$ of an acre with corn. How many tenths of an acre did he plant with each?

3. Mr. Lee has $\frac{1}{2}$ of a barrel of russet apples and $\frac{1}{3}$ of a barrel of Baldwin apples. How many twelfths of a barrel has he of each?

4. Change $\frac{1}{2}$ and $\frac{1}{3}$ to fractions having their denominators alike. $\frac{1}{2}$ and $\frac{1}{3}$. $\frac{1}{3}$ and $\frac{1}{2}$. $\frac{1}{2}$ and $\frac{1}{4}$. $\frac{1}{4}$ and $\frac{1}{2}$. $\frac{1}{3}$ and $\frac{1}{4}$. $\frac{1}{4}$ and $\frac{1}{3}$. $\frac{1}{2}$ and $\frac{1}{5}$. $\frac{1}{5}$ and $\frac{1}{2}$. $\frac{1}{3}$ and $\frac{1}{5}$. $\frac{1}{5}$ and $\frac{1}{3}$.

160. Fractions that have the same denominators are called **Similar Fractions**.

161. Fractions that have not the same denominators are called **Dissimilar Fractions**.

162. The denominator of similar fractions is called a **Common Denominator**.

163. When similar fractions are expressed in their lowest terms, they have their Least Common Denominator.

164. 1. Reduce $\frac{3}{4}$, $\frac{5}{8}$, and $\frac{11}{12}$ to similar fractions.

$$\frac{3}{4} = \frac{3 \times 6}{4 \times 6} = \frac{18}{24}$$

$$\frac{5}{8} = \frac{5 \times 3}{8 \times 3} = \frac{15}{24}$$

$$\frac{11}{12} = \frac{11 \times 2}{12 \times 2} = \frac{22}{24}$$

EXPLANATION.—Since the fractions are to be changed to other fractions having a common denominator, the terms of each fraction must be multiplied by some number which will cause them to have the same denominator.

By examining the denominators, 4, 8, and 12, it is evident that the denominators of all the fractions can be made 24, and the fractions will then be similar.

To make the denominators 24, the terms of the first fraction must be multiplied by 6, the terms of the second by 3, the terms of the third by 2. And thus the fractions are changed to the similar fractions, $\frac{18}{24}$, $\frac{15}{24}$, $\frac{22}{24}$.

Reduce to similar fractions:

$$2. \frac{3}{8}, \frac{5}{4}, \frac{5}{12}.$$

$$3. \frac{6}{5}, \frac{3}{4}, \frac{4}{15}.$$

$$4. \frac{5}{7}, \frac{3}{12}, \frac{4}{11}.$$

$$5. \frac{3}{8}, \frac{5}{15}, \frac{5}{36}.$$

$$6. \frac{3}{4}, \frac{5}{6}, \frac{7}{10}.$$

$$7. \frac{6}{5}, \frac{3}{10}, \frac{4}{5}.$$

$$8. \frac{5}{6}, \frac{3}{12}, \frac{7}{4}.$$

$$9. \frac{3}{4}, \frac{5}{21}, \frac{7}{12}.$$

$$10. \frac{3}{8}, \frac{5}{12}, \frac{3}{10}, \frac{1}{15}.$$

$$11. \frac{1}{2}, \frac{1}{3}, \frac{1}{10}, \frac{1}{15}.$$

$$12. \frac{1}{3}, \frac{5}{12}, \frac{7}{12}, \frac{5}{18}.$$

$$13. \frac{3}{4}, \frac{5}{15}, \frac{3}{25}, \frac{5}{12}.$$

14. Reduce $\frac{3}{4}$, $\frac{5}{8}$, $\frac{7}{12}$, and $\frac{9}{16}$ to similar fractions having their least common denominator.

3	3	4	12	16
2	1	4	4	16
2	1	2	2	8
	1	1	1	4

$$3 \times 2 \times 2 \times 4 = 48.$$

EXPLANATION.—The least common denominator cannot always be easily found by inspection. It may then be found as in the margin.

Since the least common denominator must be the smallest number that will contain each of the denominators, it must contain each of the prime factors of the denominators and *no other factors*. The prime factors are found as in the margin. 3 is a prime factor of 3 and 12, and consequently a factor of the least common denominator. Dividing by 3, and writing below, the quotients and numbers of which 8 is not a factor, we have 1, 4, 4, 16. Dividing by 2, and again by 2, the factors of the denominator are found to be the divisors 3, 2, 2, and the factor 4 in the last row. Their product is 48, the least common denominator. The fractions thus become $\frac{12}{48}$, $\frac{15}{48}$, $\frac{21}{48}$, $\frac{27}{48}$.

165. From these examples it is evident that:

In reducing dissimilar to similar fractions the least common denominator is found by multiplying together all the different prime factors of the given denominators. The terms of each fraction are then multiplied by such a number as will cause each fraction to have the least common denominator.

1. In finding the product of the different prime factors, each factor must be used the greatest number of times that it occurs in any of the given numbers.

2. Reduce all mixed numbers to improper fractions.

Reduce to similar fractions having the least common denominator:

- | | | |
|---|---|--|
| 15. $\frac{2}{3}, \frac{7}{12}, \frac{9}{10}$. | 20. $\frac{1}{2}, \frac{2}{3}, \frac{4}{5}$. | 25. $1\frac{1}{3}, \frac{1}{2}, 4\frac{1}{4}$. |
| 16. $\frac{4}{5}, \frac{9}{10}, \frac{1}{6}$. | 21. $\frac{5}{6}, \frac{2}{3}, \frac{7}{15}$. | 26. $4\frac{1}{4}, \frac{2}{3}, \frac{11}{12}$. |
| 17. $\frac{2}{3}, \frac{5}{12}, \frac{7}{30}$. | 22. $\frac{5}{11}, \frac{2}{3}, \frac{7}{44}$. | 27. $6\frac{1}{3}, \frac{1}{2}, \frac{1}{3}$. |
| 18. $\frac{2}{3}, \frac{4}{5}, \frac{1}{7}$. | 23. $\frac{9}{10}, \frac{2}{3}, \frac{7}{30}$. | 28. $5\frac{1}{3}, 6\frac{1}{3}, \frac{2}{3}$. |
| 19. $\frac{1}{2}, \frac{2}{3}, \frac{7}{15}$. | 24. $\frac{5}{6}, \frac{7}{30}, \frac{2}{5}$. | 29. $3\frac{1}{3}, \frac{1}{2}, \frac{1}{3}$. |

ADDITION OF FRACTIONS.

166. 1. John walked $\frac{1}{2}$ of a mile on Monday, $\frac{1}{3}$ of a mile on Tuesday, and $\frac{1}{4}$ of a mile on Wednesday. How many eighths of a mile did he walk in all?

2. How many sixths are $\frac{2}{3}, \frac{1}{2}$, and $\frac{1}{3}$?
3. How many ninths are $\frac{4}{9}, \frac{1}{3}$, and $\frac{2}{3}$?
4. How many twelfths are $\frac{3}{12}, \frac{5}{12}$, and $\frac{1}{12}$?
5. $\frac{2}{3} + \frac{1}{2} = ?$ $\frac{2}{3} + \frac{1}{3} = ?$ $\frac{3}{10} + \frac{6}{10} = ?$ $\frac{5}{11} + \frac{3}{11} = ?$ $\frac{1}{4} + \frac{1}{4} = ?$
 $\frac{1}{4} + \frac{1}{8} = ?$ $\frac{5}{18} + \frac{7}{18} = ?$ $\frac{1}{10} + \frac{3}{10} = ?$
6. A boy spent $\frac{1}{4}$ of a dollar for a book, and $\frac{1}{3}$ of a dollar for a cap. What part of a dollar did both cost?
7. A farmer sold $\frac{1}{2}$ of his sheep at one time and $\frac{1}{3}$ of them at another. What part of his sheep did he sell?
8. How many sixths are there in $\frac{1}{2}$ and $\frac{1}{3}$?
9. How many ninths are there in $\frac{1}{2}$ and $\frac{1}{3}$?

10. How many eighths are there in $\frac{1}{4}$ and $\frac{1}{8}$?
11. How many tenths are there in $\frac{1}{2}$, $\frac{1}{5}$, and $\frac{1}{10}$?
12. What kind of fractions can be added without changing their form?
13. What must be done to dissimilar fractions before they can be added?
14. Add $\frac{1}{4}$ and $\frac{1}{2}$; $\frac{1}{2}$ and $\frac{2}{3}$; $\frac{1}{3}$ and $\frac{3}{4}$; $\frac{1}{4}$ and $\frac{5}{6}$; $\frac{2}{3}$ and $\frac{1}{2}$; $\frac{1}{2}$ and $\frac{1}{3}$; $\frac{1}{3}$ and $\frac{1}{2}$; $\frac{1}{2}$ and $\frac{1}{4}$; $\frac{1}{4}$ and $\frac{1}{3}$.
15. Add $\frac{1}{2}$ and $\frac{1}{3}$; $\frac{1}{3}$ and $\frac{1}{4}$; $\frac{1}{4}$ and $\frac{1}{5}$; $\frac{1}{5}$ and $\frac{1}{6}$; $\frac{1}{6}$ and $\frac{1}{7}$.
16. Add $\frac{1}{2}$ and $\frac{3}{5}$; $\frac{1}{2}$ and $\frac{4}{5}$; $\frac{3}{5}$ and $\frac{4}{5}$; $\frac{3}{5}$ and $\frac{2}{3}$; $\frac{2}{3}$ and $\frac{4}{5}$.
17. I paid $\$ \frac{1}{2}$ for milk and $\$ \frac{1}{10}$ for lettuce. How much did I pay for both?
18. James paid $\$ \frac{2}{5}$ for a fishing-rod and $\$ \frac{1}{10}$ for a line. What part of a dollar did both cost him?
19. Lucy worked $\frac{1}{2}$ of a day at her lessons and spent $\frac{1}{3}$ of a day at a picnic. What part of the day did she spend at both?
20. A farmer's son sold $\frac{1}{2}$ dozen eggs to one man, $\frac{1}{4}$ dozen to another, and $\frac{1}{3}$ dozen to another. How many dozen did he sell in all?
21. A lad was presented with some money on his birthday. The next day he spent $\frac{1}{3}$ of it, the day following, $\frac{1}{2}$ of it, and on the third day he spent $\frac{1}{10}$ of it. What part of the money did he spend?

WRITTEN EXERCISES.

- 167.** 1. What is the sum of $\frac{5}{8}$, $\frac{3}{4}$, and $\frac{1}{2}$?

$$\frac{5}{8} = \frac{25}{40}$$

$$\frac{3}{4} = \frac{30}{40}$$

$$\frac{1}{2} = \frac{20}{40}$$

$$\frac{5}{8} + \frac{3}{4} + \frac{1}{2} = \frac{27}{40}$$

$$\frac{5}{8} + \frac{3}{4} + \frac{1}{2} = 2\frac{7}{40}$$

$$\frac{5}{8} + \frac{3}{4} + \frac{1}{2} = 2\frac{7}{40}$$

EXPLANATION.— Since the fractions are not similar, they must be changed to similar fractions before they are added.

The least common denominator of the given fractions is 40, and $\frac{5}{8} = \frac{25}{40}$, $\frac{3}{4} = \frac{30}{40}$, and $\frac{1}{2} = \frac{20}{40}$. Hence

the sum of the given fractions will be the sum of $\frac{25}{40}$, $\frac{30}{40}$, and $\frac{20}{40}$, which is $\frac{75}{40}$, or $2\frac{7}{40}$.

168. From the above solutions, it is evident that:

In adding fractions, dissimilar fractions are first changed to similar ones, then their numerators are added, and the sum is placed over the common denominator.

1. If the sum is an improper fraction, it should be reduced to an integer or mixed number.
2. When there are mixed numbers or integers to be added, the integers and fractions should be added separately and then those results added.

Find the sum of the following:

- | | |
|--|---|
| 2. $\frac{5}{6}$, $\frac{7}{12}$, and $\frac{1}{4}$. | 10. $6\frac{4}{5}$, $2\frac{3}{4}$, and $4\frac{4}{15}$. |
| 3. $\frac{3}{10}$, $\frac{7}{15}$, and $\frac{1}{5}$. | 11. $3\frac{1}{4}$, $1\frac{8}{11}$, and $5\frac{1}{2}$. |
| 4. $\frac{2}{3}$, $\frac{4}{5}$, and $\frac{7}{15}$. | 12. $4\frac{1}{4}$, $3\frac{2}{3}$, and $8\frac{7}{18}$. |
| 5. $\frac{3}{5}$, $\frac{7}{20}$, and $\frac{1}{10}$. | 13. $5\frac{1}{5}$, $10\frac{1}{20}$, and $\frac{1}{8}$. |
| 6. $\frac{5}{18}$, $\frac{8}{20}$, and $\frac{1}{6}$. | 14. $6\frac{1}{4}$, $4\frac{1}{5}$, and $1\frac{1}{21}$. |
| 7. $\frac{3}{4}$, $\frac{5}{12}$, and $\frac{1}{4}$. | 15. $12\frac{1}{4}$, $8\frac{2}{3}$, and $2\frac{3}{20}$. |
| 8. $\frac{4}{5}$, $\frac{9}{21}$, and $\frac{2}{3}$. | 16. $9\frac{4}{15}$, $\frac{7}{5}$, and $11\frac{1}{24}$. |
| 9. $\frac{3}{20}$, $\frac{7}{12}$, and $\frac{5}{6}$. | 17. $7\frac{7}{40}$, $3\frac{2}{60}$, and $6\frac{1}{20}$. |

18. A farmer sold a load of hay for $\$8\frac{3}{4}$, a load of oats for $\$22\frac{5}{8}$, and a load of wheat for $\$53\frac{3}{4}$. How much did he receive for all?

19. James picked $5\frac{2}{3}$ bushels of apples from one tree, $7\frac{1}{2}$ bushels from another, $6\frac{8}{10}$ bushels from another, and $8\frac{1}{2}$ bushels from another. How many bushels did he pick in all?

20. Three barrels of sugar contained respectively $225\frac{1}{2}$ pounds, $232\frac{5}{12}$ pounds, and $240\frac{1}{20}$ pounds. What was the weight of the whole?

21. A school-room is $32\frac{1}{2}$ feet long and $29\frac{1}{4}$ feet wide. What is the distance around the room?

22. An express train starts from Boston at $1\frac{3}{4}$ P.M., reaches Springfield $2\frac{3}{4}$ hours later, and Albany $4\frac{3}{10}$ hours later still. At what o'clock does it arrive in Albany?

SUBTRACTION OF FRACTIONS.

169. 1. Bertie earned $\$ \frac{1}{2}$ and spent $\$ \frac{1}{3}$. How much had he left?

2. Susie had $\frac{1}{2}$ of a pound of candy, but she ate $\frac{1}{3}$ of a pound. How much had she left?

3. There were $\frac{1}{2}$ of a bushel of oats in a bin. If $\frac{1}{3}$ of a bushel was taken out, how much remained?

4. $\frac{5}{7} - \frac{2}{7} = ?$ $\frac{7}{12} - \frac{1}{12} = ?$ $\frac{5}{8} - \frac{3}{8} = ?$ $\frac{9}{10} - \frac{3}{10} = ?$
 $\frac{11}{16} - \frac{5}{16} = ?$ $\frac{1}{2} - \frac{1}{3} = ?$ $\frac{1}{2} - \frac{1}{3} = ?$

5. If I have $\$ \frac{3}{4}$ and give away $\$ \frac{1}{4}$, what part of a dollar shall I have left?

6. A man who had $\frac{1}{2}$ of a cord of wood sold $\frac{1}{3}$ of a cord. What part of a cord had he left?

7. One pitcher held $\frac{1}{2}$ of a quart of milk, and another held $\frac{1}{3}$ of a quart. How much more did the one hold than the other?

8. $\frac{1}{2} - \frac{1}{4} = ?$ $\frac{3}{4} - \frac{1}{2} = ?$ $\frac{1}{2} - \frac{1}{6} = ?$ $\frac{5}{6} - \frac{1}{2} = ?$ $\frac{1}{2} - \frac{1}{3} = ?$
 $\frac{7}{8} - \frac{1}{2} = ?$

9. What kind of fractions can be subtracted without changing their form?

10. What must be done to dissimilar fractions before they can be subtracted?

11. $1 - \frac{2}{3} = ?$ $1 - \frac{4}{5} = ?$ $1 - \frac{3}{4} = ?$ $1 - \frac{5}{6} = ?$ $1 - \frac{7}{8} = ?$

12. $\frac{1}{2} - \frac{1}{3} = ?$ $\frac{1}{3} - \frac{1}{4} = ?$ $\frac{1}{4} - \frac{1}{5} = ?$ $\frac{1}{5} - \frac{1}{6} = ?$ $\frac{1}{6} - \frac{1}{7} = ?$
 $\frac{1}{7} - \frac{1}{8} = ?$

13. $\frac{1}{2} - \frac{1}{3} = ?$ $\frac{1}{3} - \frac{1}{4} = ?$ $\frac{1}{4} - \frac{1}{5} = ?$ $\frac{1}{5} - \frac{1}{6} = ?$ $\frac{1}{6} - \frac{1}{7} = ?$
 $\frac{1}{7} - \frac{1}{8} = ?$

14. A lad earned one day $\$ \frac{1}{2}$ and spent $\$ \frac{1}{3}$. How much had he left?

15. A mechanic's wages were $\$ 3$ per day, but he paid $\$ \frac{3}{4}$ per day for his board. How much did he save daily?

16. A boy agreed to work for a man 5 days, but lost $1\frac{1}{2}$ days by sickness. How many days did he work?

17. A boy raised vegetables which he sold for \$3. The cost of seed and other expenses were \$1 $\frac{1}{10}$. How much was his net profit?

18. A man bought a hat for \$2 $\frac{1}{2}$, a vest for \$2 $\frac{3}{4}$, and a linen coat for \$1 $\frac{1}{2}$. How much change should he receive from the merchant, if he paid for them with a ten-dollar bill?

WRITTEN EXERCISES.

170. 1. What is the difference between $1\frac{5}{8}$ and $1\frac{7}{12}$?

EXPLANATION. — Since the fractions are not similar, they must be made similar before they can be subtracted. The least common denominator of the given fractions is 48; $1\frac{5}{8} = 1\frac{45}{48}$, and $1\frac{7}{12} = 1\frac{28}{48}$. Hence, the difference between the given fractions is the difference between $\frac{45}{48}$ and $\frac{28}{48}$, which is $\frac{17}{48}$.

2. What is the difference between $15\frac{9}{12}$ and $8\frac{5}{6}$?

EXPLANATION. — Since the numbers are composed of integers and fractions, each may be subtracted separately. Reducing the fractions to similar fractions, it is evident that $\frac{9}{12}$ cannot be subtracted from $\frac{5}{6}$, hence 1, taken from 15, is united with $\frac{5}{6}$. 1, or $\frac{1}{2} + \frac{1}{3} = \frac{5}{6}$. Subtracting $8\frac{5}{6}$ from $14\frac{9}{12}$, there is a remainder of $6\frac{1}{2}$.

171. From the solution of the above examples it is seen that:

In subtracting fractions, dissimilar fractions are changed to similar ones, then their numerators are subtracted, and the difference is written over the common denominator.

When there are integers or mixed numbers to be subtracted, the integers and fractions should be subtracted separately.

	3.	4.	5.	6.	7.	8.	9.
Out of	$1\frac{2}{3}$	$\frac{7}{8}$	$1\frac{5}{8}$	$1\frac{1}{8}$	$1\frac{5}{8}$	$1\frac{1}{12}$	$1\frac{1}{12}$
Take	$\frac{5}{6}$	$\frac{5}{6}$	$1\frac{3}{4}$	$1\frac{1}{2}$	$\frac{5}{6}$	$\frac{5}{6}$	$1\frac{1}{12}$

	10.	11.	12.	13.	14.	15.	16.
From	$\frac{3}{5}$	$\frac{6}{5}$	$\frac{17}{4}$	9	20	15	26
Take	$\frac{3}{10}$	$\frac{3}{2}$	$\frac{11}{6}$	$2\frac{5}{11}$	$7\frac{3}{40}$	$10\frac{9}{15}$	$7\frac{1}{4}$
	17.	18.	19.	20.	21.	22.	23.
From	$6\frac{1}{2}$	$8\frac{5}{12}$	$11\frac{1}{2}$	$16\frac{1}{2}$	$20\frac{5}{14}$	$31\frac{9}{15}$	$63\frac{1}{5}$
Take	$2\frac{3}{7}$	$2\frac{5}{24}$	$5\frac{1}{2}$	$10\frac{1}{6}$	$16\frac{1}{12}$	$20\frac{5}{28}$	$39\frac{3}{5}$

24. A box of soap weighed $75\frac{1}{2}$ pounds. The weight of the box alone was $3\frac{5}{12}$ pounds. How much did the soap weigh?

25. From a farm of 120 acres there were sold $36\frac{1}{4}$ acres. How much was left?

26. A flag-staff $50\frac{1}{10}$ feet high was broken off by a storm, so that it measured $43\frac{3}{4}$ feet. How much was broken off?

27. A grocer had $128\frac{1}{2}$ pounds of sugar in one barrel, and $98\frac{1}{4}$ pounds in another. How much more was in the one barrel than the other?

28. A clerk earned \$75 per month. His expenses during that time were as follows: board, \$22 $\frac{1}{2}$; washing, \$5 $\frac{1}{2}$; and other expenses \$16 $\frac{1}{2}$. How much did he save per month?

29. A man spent $\frac{3}{10}$ of his money for a house, $\frac{1}{2}$ for furniture, and $\frac{1}{8}$ for horses and carriages. What part of his money had he left?

30. The sum of two numbers is $11\frac{5}{8}$. One of the numbers is $8\frac{1}{4}$. What is the other number?

31. From a web of cotton cloth containing $58\frac{7}{16}$ yards, $8\frac{1}{2}$ yards were cut off at one time, and $15\frac{3}{8}$ yards at another time. How many yards remained?

32. Mr. Allen left $\frac{1}{2}$ of his property to his wife, $\frac{3}{10}$ to his sons, $\frac{1}{4}$ to his daughter, and the remainder to charitable institutions. What part of his property was left to charitable institutions?

Find the value of the following :

- | | |
|--|--|
| 33. $\frac{5}{8} + \frac{3}{12} + \frac{1}{24} + \frac{5}{16}$. | 40. $3\frac{1}{2} + 4\frac{2}{3} + 3\frac{3}{4} - 1\frac{1}{2}$. |
| 34. $\frac{3}{4} + \frac{5}{8} + \frac{7}{12} + \frac{11}{16}$. | 41. $6\frac{3}{5} - 4\frac{2}{3} + 3\frac{1}{10} - 1\frac{3}{10}$. |
| 35. $\frac{4}{5} + \frac{7}{10} - \frac{1}{4} - \frac{1}{3}$. | 42. $8\frac{5}{6} + 2\frac{7}{8} - 3\frac{1}{5} - 3\frac{1}{2}$. |
| 36. $\frac{6}{15} + \frac{3}{8} - \frac{1}{10} - \frac{1}{50}$. | 43. $2\frac{2}{3} + 3\frac{3}{4} - 1\frac{1}{8} + 2\frac{5}{8}$. |
| 37. $\frac{2}{25} - \frac{3}{8} + \frac{5}{7} - \frac{3}{15}$. | 44. $5\frac{1}{3} - 2\frac{2}{3} - 1\frac{1}{2} + 4\frac{3}{15}$. |
| 38. $\frac{5}{21} - \frac{3}{42} + \frac{6}{7} - \frac{1}{6}$. | 45. $2\frac{3}{5} + 6\frac{5}{11} - 3\frac{3}{10} - 1\frac{4}{15}$. |
| 39. $\frac{1}{16} - \frac{2}{3} + \frac{3}{4} - \frac{7}{12}$. | 46. $6\frac{5}{7} + 2\frac{3}{4} - 1\frac{5}{8} + 2\frac{3}{5}$. |

MULTIPLICATION OF FRACTIONS.

172. To multiply a fraction by an integer.

1. At $\$ \frac{2}{3}$ a yard, what will 2 yards of cloth cost?
2. If a family uses $\frac{2}{5}$ of a pound of butter each day, how much will it use in 2 days?
3. If a man earns $\$ \frac{3}{10}$ an hour, how much will he earn in 3 hours?
4. How much is 3 times $\frac{2}{7}$? 4 times $\frac{3}{5}$? 5 times $\frac{1}{11}$? 3 times $\frac{8}{15}$? 2 times $\frac{4}{3}$? 7 times $\frac{2}{15}$? 6 times $\frac{3}{20}$?
5. How, then, may a fraction be multiplied by any integer?
6. If I gave $\frac{1}{2}$ of an orange to each of 5 boys, how many halves did I give away? How many whole oranges was that?
7. At $\$ \frac{3}{4}$ a yard, what will 2 yards of velvet ribbon cost?
8. Mary's mother has 4 tumblers of jelly, each holding $\frac{2}{3}$ of a pint. How many pints of jelly has she?
9. 5 times $\frac{3}{8}$ = ? 2 times $\frac{5}{6}$ = ? 3 times $\frac{3}{4}$ = ? 4 times $\frac{2}{3}$ = ? 2 times $\frac{4}{5}$ = ? 3 times $\frac{5}{6}$ = ? 4 times $\frac{3}{4}$ = ? 6 times $\frac{3}{10}$ = ?
10. How many are 4 times $1\frac{1}{2}$? 3 times $2\frac{1}{2}$? 5 times $3\frac{1}{2}$?
11. Multiply $4\frac{1}{4}$ by 5; $5\frac{1}{4}$ by 3; $9\frac{1}{4}$ by 8; $6\frac{1}{4}$ by 9.
12. How much is 5 times $\frac{4}{15}$? Express the result in its smallest terms. How may this result be obtained from $\frac{1}{15}$?

13. How much is 3 times $\frac{7}{12}$? Express the result in its smallest terms. How may this result be obtained from $\frac{7}{12}$?

14. In what other way, therefore, may we sometimes multiply a fraction by an integer? Give both ways.

15. If a bookseller sells 15 books at $\$ \frac{7}{30}$ apiece, how much will he get for them?

16. When flour sells at $\$ 4\frac{1}{2}$ per barrel, how much will 9 barrels cost?

17. A clerk's wages were $\$ 10\frac{1}{4}$ per week. How much would he earn at that rate in 8 weeks?

18. If the wages of a laborer are $\$ 1\frac{3}{4}$ per day, how much does he earn in 10 days?

19. A good walker can walk $4\frac{1}{2}$ miles per hour. How far can he walk at the same rate in 8 hours?

173. It has been discovered, therefore, that:

A fraction can be multiplied by an integer by multiplying the numerator or dividing the denominator by the integer.

WRITTEN EXERCISES.

174. 1. Multiply $\frac{11}{24}$ by 8.

$$\frac{11}{24} \times 8 = \frac{11 \times 8}{24} = \frac{88}{24} = 3\frac{4}{24} \text{, or } 3\frac{1}{6}.$$

$$\text{Or, } \frac{11}{24} \times 8 = \frac{11}{24 + 8} = \frac{11}{3} = 3\frac{1}{3}.$$

EXPLANATION. — 8 times $\frac{11}{24}$ is $\frac{88}{24}$, or $3\frac{4}{24}$, or $3\frac{1}{6}$. Or, since dividing the denominator multiplies the fraction, 8 times $\frac{11}{24}$ is $\frac{11}{3}$, or $3\frac{1}{3}$.

Multiply:

2. $\frac{8}{25}$ by 5.

7. $\frac{6}{5}$ by 16.

12. $3\frac{3}{8}$ by 6.

3. $\frac{5}{22}$ by 4.

8. $\frac{7}{10}$ by 15.

13. $6\frac{5}{8}$ by 12.

4. $\frac{11}{4}$ by 6.

9. $\frac{11}{2}$ by 14.

14. $10\frac{7}{12}$ by 15.

5. $\frac{11}{3}$ by 5.

10. $\frac{8}{15}$ by 20.

15. $14\frac{3}{4}$ by 18.

6. $\frac{17}{6}$ by 9.

11. $\frac{8}{18}$ by 23.

16. $16\frac{5}{8}$ by 8.

17. What will 5 pounds of cheese cost at $14\frac{1}{2}\text{¢}$ per pound?
18. If a train goes at the rate of $20\frac{1}{2}$ miles an hour, how far will it go in 12 hours?
19. When potatoes are worth $66\frac{2}{3}\text{¢}$ a bushel, what must I pay for 20 bushels?
20. How much will 15 tons of coal cost at $\$6\frac{7}{10}$ a ton?
21. How many feet are there in 10 rods, since there are $16\frac{1}{2}$ feet in 1 rod?
22. A boy can chop $\frac{1}{8}$ of a cord of wood a day. How much can he chop in 3 weeks, or 18 days?
23. If a man works $8\frac{1}{2}$ hours per day, how many hours will he work in 20 days?
24. If the railroad fare between two places is $\$6\frac{4}{5}$, what will the fare of 10 persons cost?

175. To multiply an integer by a fraction.

1. James had 12 chickens, but sold $\frac{1}{3}$ of them. How many did he sell? What is $\frac{1}{3}$ of 12 chickens?
2. What will $\frac{1}{4}$ of a pound of raisins cost at 16¢ per pound? What is $\frac{1}{4}$ of 16¢ ?
3. Mr. Bird's horse can trot 5 miles in 20 minutes. How many minutes will it take him to go 1 mile? What is $\frac{1}{5}$ of 20? $\frac{1}{5}$ of 20?
4. What is $\frac{1}{2}$ of 24? $\frac{1}{2}$ of 14? $\frac{1}{2}$ of 18? $\frac{1}{2}$ of 28?
5. Two yards of ribbon were divided equally among 3 girls. How much did each girl get?
6. If 1 yard of braid is worth 3¢ , what is $\frac{1}{4}$ of a yard worth? What is $\frac{1}{4}$ of 4¢ ?
7. Four quarts of milk filled 5 pitchers of equal size. How much did each pitcher hold? What is $\frac{1}{5}$ of 4? $\frac{1}{5}$ of 4? $\frac{1}{5}$ of 4?
8. What is $\frac{1}{2}$ of 3? $\frac{1}{2}$ of 2? $\frac{1}{10}$ of 7? $\frac{1}{2}$ of 3? $\frac{1}{2}$ of 5?
9. What is $\frac{1}{3}$ of 4? $\frac{1}{3}$ of 4? $\frac{1}{3}$ of 7? $\frac{1}{3}$ of 5? $\frac{1}{3}$ of 5? $\frac{1}{3}$ of 9? $\frac{1}{3}$ of 8? $\frac{1}{3}$ of 8? $\frac{1}{3}$ of 15? $\frac{1}{3}$ of 12? $\frac{1}{3}$ of 12?

10. A coat cost \$12, and a pair of trousers and a vest $\frac{3}{4}$ as much. How much did the suit cost?

11. A schoolboy paid $\frac{1}{10}$ of his money for a pencil, $\frac{3}{4}$ of it for a grammar, and $\frac{1}{2}$ of it for a penholder. If he had 40¢ before he made his purchases, how much had he left?

12. A lesson in arithmetic contained 30 examples. Henry solved $\frac{2}{3}$ of them, and his sister solved $\frac{9}{10}$ of them. How many did each solve?

13. The month of February has usually 28 days. If, in 1891, $\frac{2}{3}$ of them were stormy, how many were pleasant?

14. A paymaster paid his men only $\frac{2}{3}$ of their monthly wages. If each man earned \$35, how much did he pay to each?

176. From these examples it is seen that:

Multiplying a number by a fraction is finding a fractional part of the number.

Thus, to multiply 12 by $\frac{2}{3}$ we simply find $\frac{2}{3}$ of 12.

WRITTEN EXERCISES.

177. 1. Multiply 13 by $\frac{4}{5}$, or find $\frac{4}{5}$ of 13.

EXPLANATION. — To multiply $13 \times \frac{4}{5} = \frac{13 \times 4}{5} = \frac{52}{5}$, or $10\frac{2}{5}$. 13 by $\frac{4}{5}$ is to find $\frac{4}{5}$ of 13. $\frac{1}{5}$ of 13 is $\frac{13}{5}$, and $\frac{4}{5}$ of 13 is 4 times $\frac{13}{5}$, which is $\frac{52}{5}$, or $10\frac{2}{5}$.

When the multiplier is a mixed number, multiply by the integer and fraction separately, and add the results.

What is:

2. $\frac{2}{3}$ of 15?

3. $\frac{4}{5}$ of 18?

4. $\frac{8}{11}$ of 34?

5. $\frac{7}{8}$ of 24?

6. $\frac{7}{16}$ of 28?

7. $\frac{9}{20}$ of 45?

Multiply:

8. 54 by $\frac{7}{6}$.

9. 35 by $\frac{5}{6}$.

10. 48 by $\frac{11}{10}$.

11. 64 by $\frac{13}{8}$.

12. 81 by $\frac{17}{18}$.

13. 60 by $\frac{8}{5}$.

Multiply:

14. 18 by $1\frac{1}{4}$.

15. 25 by $3\frac{2}{3}$.

16. 42 by $5\frac{1}{2}$.

17. 34 by $7\frac{1}{3}$.

18. 29 by $\frac{3}{4}$.

19. 56 by $9\frac{1}{4}$.

20. When corn is worth 75¢ a bushel, what is the value of $\frac{1}{4}$ of a bushel?
21. There are 125 acres in Mr. Swift's farm. How many acres are there in $\frac{1}{5}$ of the farm?
22. The expenses of a foundry for one month were \$583. What will $\frac{1}{3}$ of the expenses be?
23. What will 6 $\frac{1}{2}$ yards of silk cost at 88¢ a yard?
24. The average rate per hour of a very fast express train is 52 miles. How far will it go in 8 $\frac{7}{10}$ hours?
25. What will be the cost of 8 $\frac{1}{2}$ pounds of tea at 30¢ a pound, 6 $\frac{1}{2}$ pounds of sugar at 5¢ a pound, and 13 $\frac{1}{2}$ pounds of raisins at 16¢ a pound?
26. What will be the cost of 6 $\frac{1}{2}$ dozen arithmetics at \$8 per dozen, 5 $\frac{7}{12}$ dozen grammars at \$6 per dozen, and 6 $\frac{1}{2}$ dozen geographies at \$9 per dozen?

178. To multiply a fraction by a fraction.

1. If $\frac{1}{2}$ of an apple is divided into 2 equal parts, what part of the apple will each part be? How much is $\frac{1}{2}$ of $\frac{1}{2}$ of an apple?
2. If $\frac{1}{3}$ of a yard is divided into 2 equal parts, what part of a yard will each part be? How much is $\frac{1}{3}$ of $\frac{1}{2}$?
3. If $\frac{1}{4}$ of a pie is divided into 2 equal parts, what part of the pie will each part be? How much is $\frac{1}{4}$ of $\frac{1}{2}$?
4. If $\frac{1}{5}$ of a pie is divided into 3 equal parts, what part of the pie will each part be? How much is $\frac{1}{5}$ of $\frac{1}{3}$?
5. How much is $\frac{1}{2}$ of $\frac{1}{2}$? $\frac{1}{3}$ of $\frac{1}{2}$? $\frac{1}{4}$ of $\frac{1}{2}$? $\frac{1}{5}$ of $\frac{1}{2}$? $\frac{1}{6}$ of $\frac{1}{2}$? $\frac{1}{7}$ of $\frac{1}{2}$? $\frac{1}{8}$ of $\frac{1}{2}$? $\frac{1}{9}$ of $\frac{1}{2}$? $\frac{1}{10}$ of $\frac{1}{2}$?
6. How much is $\frac{1}{2}$ of $\frac{1}{2}$? Since $\frac{1}{2}$ of $\frac{1}{2}$ is $\frac{1}{10}$, how much is $\frac{2}{3}$ of $\frac{1}{2}$? $\frac{3}{4}$ of $\frac{1}{2}$? $\frac{4}{5}$ of $\frac{1}{2}$?
7. A man having $\frac{1}{2}$ of an acre of land sold $\frac{1}{3}$ of it. What part of an acre did he sell?
8. If a yard of crape costs \$ $\frac{1}{2}$, what part of a dollar will $\frac{1}{2}$ of a yard cost?

9. I bought $\frac{1}{2}$ of a bushel of beans and planted $\frac{2}{3}$ of them. What part of a bushel did I plant?

10. How much is $\frac{1}{2}$ of $\frac{1}{2}$? Since $\frac{1}{2}$ of $\frac{1}{2}$ is $\frac{1}{4}$, how much is $\frac{1}{2}$ of $\frac{2}{3}$? $\frac{1}{2}$ of $\frac{4}{3}$? $\frac{1}{2}$ of $\frac{3}{2}$? $\frac{2}{3}$ of $\frac{2}{3}$?

11. When cloth is worth $\$ \frac{7}{8}$ a yard, what will $\frac{1}{2}$ of a yard cost? What will $\frac{3}{4}$ of a yard cost?

12. Mr. Stone, having a lot containing $\frac{2}{3}$ of an acre, sold $\frac{1}{2}$ of it. What part of an acre did he sell? If he had sold $\frac{2}{3}$ of it, what part of an acre would he have sold?

13. If a boy who had $\$ \frac{4}{5}$ spent $\frac{1}{2}$ of it for candy, what part of a dollar did he spend? If he had spent $\frac{2}{3}$ of it, what part of a dollar would he have spent?

14. Mr. Stearns, who owned $\frac{3}{4}$ of a store, sold $\frac{2}{3}$ of his share. What part of the whole store did he sell?

15. What is $\frac{2}{3}$ of $\frac{3}{4}$? $\frac{2}{3}$ of $\frac{4}{3}$? $\frac{2}{3}$ of $\frac{3}{2}$? $\frac{3}{4}$ of $\frac{3}{2}$?

WRITTEN EXERCISES.

179. 1. Multiply $\frac{5}{8}$ by $\frac{4}{7}$, or find $\frac{4}{7}$ of $\frac{5}{8}$.

EXPLANATION.—To multiply $\frac{5}{8}$ by $\frac{4}{7}$, is to find $\frac{4}{7}$ of $\frac{5}{8}$. $\frac{4}{7}$ of $\frac{1}{8}$ is $\frac{1}{14}$ ($\frac{1}{7} \times \frac{1}{8}$); then, $\frac{1}{7}$ of $\frac{5}{8}$ will be $(\frac{1}{7} \times \frac{5}{8}) \frac{5}{56}$; and $\frac{4}{7}$ of $\frac{5}{8}$ will be 4 times $\frac{5}{56}$, or $\frac{20}{56}$, or $\frac{5}{14}$.

180. From the above it is seen that:

In multiplying a fraction by a fraction, the numerators are multiplied together for the numerator of the product and the denominators, for its denominator.

1. Reduce any mixed numbers to improper fractions.
2. Integers may be expressed as fractions by writing 1 as a denominator. Thus, 4 may be written $\frac{4}{1}$.
3. When possible use cancellation.
4. The word *of* between the fractions is equivalent to the sign of multiplication. Such expressions are sometimes called *Compound Fractions*. Thus, $\frac{1}{2}$ of $\frac{2}{3}$ is equal to $\frac{1}{2} \times \frac{2}{3}$.

Find :

2. $\frac{2}{3}$ of $\frac{5}{6}$.

3. $\frac{4}{5}$ of $\frac{7}{8}$.

4. $\frac{2}{3}$ of $\frac{1}{4}$.

5. $\frac{5}{6}$ of $\frac{3}{4}$.

6. $\frac{1}{2}$ of $\frac{2}{3}$.

7. $\frac{3}{4}$ of $\frac{2}{3}$.

8. $\frac{1}{3}$ of $\frac{3}{2}$.

9. $\frac{1}{3}$ of $\frac{2}{3}$.

10. $\frac{8}{11}$ of $\frac{3}{5}$.

11. $\frac{7}{12}$ of $\frac{7}{12}$.

12. $\frac{9}{14}$ of $\frac{1}{14}$.

13. $\frac{1}{2}$ of $\frac{1}{12}$.

Find :

Multiply :

14. $\frac{2}{3}$ of $4\frac{1}{2}$.

15. $\frac{5}{6}$ of $10\frac{1}{2}$.

16. $\frac{7}{10}$ of $8\frac{5}{8}$.

17. $\frac{1}{3}$ of $7\frac{3}{4}$.

18. $\frac{2}{3}$ of $\frac{1}{2}$ by $\frac{5}{6}$.

19. $\frac{6}{5}$ of $\frac{7}{11}$ by $3\frac{1}{4}$.

20. $5\frac{1}{2}$ by $\frac{2}{3}$ of $3\frac{1}{2}$.

21. $6\frac{1}{2}$ by $\frac{4}{5}$ of $4\frac{1}{4}$.

Find the value of :

22. $\frac{2}{3} \times \frac{2}{3} \times \frac{2}{3} \times \frac{2}{3}$.

23. $\frac{2}{3} \times \frac{2}{3} \times \frac{1}{2} \times \frac{1}{2}$.

24. $\frac{2}{3} \times \frac{2}{3} \times \frac{2}{3} \times \frac{2}{3}$.

25. $2\frac{1}{2} \times \frac{2}{3} \times 1\frac{1}{3} \times \frac{2}{3}$.

26. There are $16\frac{1}{2}$ feet in a rod. How many feet are there in $\frac{2}{3}$ of a rod? In $3\frac{1}{2}$ rods?

27. When hay is worth $\$20\frac{1}{2}$ per ton, how much will $\frac{7}{8}$ of a ton cost?

28. At $\$6\frac{1}{2}$ a ton, what must I pay for $8\frac{1}{4}$ tons of coal?

29. A man purchased $\frac{1}{3}$ of $420\frac{1}{2}$ acres of land and then sold $\frac{2}{3}$ of what he had bought. How many acres did he sell?

30. At an auction sale a cow was sold for $\$39\frac{1}{2}$, and a horse for $3\frac{1}{2}$ times as much as the cow. For how much was the horse sold?

31. If a square foot of land is worth $16\frac{1}{2}$, what is the value of $10\frac{1}{2}$ square feet?

32. The cloth of Ruth's dress cost $\$8\frac{1}{2}$. The making cost $\frac{1}{2}$ as much, the trimming cost $\frac{7}{10}$ as much as the making, and the linings cost $\frac{3}{4}$ as much as the trimmings. What did the linings cost? What was the whole cost of the dress?

DIVISION OF FRACTIONS.

181. To divide a fraction by an integer.

1. If $\frac{4}{5}$ of an orange is divided equally between 2 girls, what part of the orange will each girl have? How much is $\frac{4}{5} \div 2$?
2. A man divided $\frac{4}{5}$ of an acre into 3 equal lots. How large was each? How much is $\frac{4}{5} \div 3$?
3. If a horse ate $\frac{8}{5}$ of a bushel of oats in 4 days, how much did he eat each day? How much is $\frac{8}{5} \div 4$?
4. In dividing a fraction by an integer, what part of the fraction is divided?
5. If $\$ \frac{1}{2}$ is divided equally between 2 boys, what part of a dollar will each boy get? How much is $\frac{1}{2} \div 2$?
6. When 3 pounds of sugar can be bought for $\$ \frac{1}{4}$, what is the price per pound? How much is $\frac{1}{4} \div 3$?
7. If $\frac{1}{2}$ of a ship is owned equally by 5 men, what part of the ship is owned by each? How much is $\frac{1}{2} \div 5$?
8. In what other way, besides dividing the numerator, can a fraction be divided by an integer? State both ways in which a fraction may be divided by an integer.
9. If 3 dozen oranges can be bought for $\$ \frac{6}{7}$, what do they cost per dozen?
10. If $\frac{3}{4}$ of a pound of tea is divided equally among 3 persons, what part of a pound will each receive?
11. If $\frac{4}{5}$ of a bushel of walnuts fills 4 bags of equal size, what part of a bushel does each bag hold?
12. Mrs. Jay filled 5 tumblers with $\frac{1}{8}$ of a gallon of jelly. How much did each tumbler hold?
13. Divide $\frac{2}{3}$ by 2; $\frac{9}{10}$ by 2; $\frac{9}{10}$ by 3; $\frac{8}{11}$ by 4; $\frac{1}{8}$ by 5.
14. Divide $\frac{3}{4}$ by 2; $\frac{7}{8}$ by 3; $\frac{5}{6}$ by 4; $\frac{4}{5}$ by 5; $\frac{7}{6}$ by 6.
15. If 3 yards of cloth cost $\$ 1\frac{1}{4}$, what does 1 yard cost?
16. When 5 bushels of potatoes cost $\$ 4\frac{1}{2}$, what does a bushel cost?

17. If 3 dozen oranges cost $\$ \frac{4}{5}$, what does 1 dozen cost?
 18. If 4 brooms are worth $\$ \frac{8}{15}$, how much are they worth apiece?
 19. When I can buy 4 dozen eggs for $\$ \frac{4}{15}$, what is the price per dozen?
 20. If 5 yards of gingham sell for $\$ \frac{4}{5}$, what is the price per yard?
 21. If 6 pounds of tea are worth $\$ 1 \frac{1}{4}$, what is a pound worth?
 22. When 5 bushels of wheat sell for $\$ 4 \frac{1}{4}$, what is the price per bushel?
 23. James earned $\$ 6 \frac{2}{5}$ in 6 days. What were his daily wages?
 24. A boy divided $6 \frac{1}{2}$ dozen marbles equally among his 5 sisters. How many did each receive?
- 182.** It is apparent from the solution of the above examples that:

A fraction may be divided by an integer by dividing the numerator or multiplying the denominator of the fraction by the integer.

WRITTEN EXERCISES.

- 183.** 1. Divide $\frac{20}{21}$ by 5, or find $\frac{1}{5}$ of $\frac{20}{21}$.

$$\frac{20}{21} \div 5 = \frac{20+5}{21} = \frac{4}{21}.$$

EXPLANATION. — Since dividing the numerator of a fraction divides the fraction, the fraction $\frac{20}{21}$ may be divided by 5 by dividing the numerator by 5. Hence the result is $\frac{4}{21}$.

2. Divide $\frac{5}{12}$ by 6, or find $\frac{1}{6}$ of $\frac{5}{12}$.

$$\frac{5}{12} \div 6 = \frac{5}{12 \times 6} = \frac{5}{72}.$$

EXPLANATION. — Since multiplying the denominator of a fraction divides the fraction, the fraction $\frac{5}{12}$ may be divided by 6 by multiplying the denominator by 6. Hence the result is $\frac{5}{72}$.

Divide :

3. $\frac{1}{2}$ by 4.

7. $\frac{11}{8}$ by 12.

11. $\frac{17}{4}$ by 23.

4. $\frac{4}{3}$ by 5.

8. $\frac{1}{2}$ by 16.

12. $\frac{1}{2}$ by 28.

5. $\frac{2}{3}$ by 8.

9. $\frac{3}{4}$ by 18.

13. $\frac{1}{3}$ by 35.

6. $\frac{1}{4}$ by 7.

10. $\frac{1}{2}$ by 15.

14. $\frac{1}{4}$ by 42.

15. Divide $11\frac{3}{4}$ by 4.

$$11\frac{3}{4} = \frac{47}{4}$$

$$\frac{47}{4} \div 4 = \frac{47}{16} = 2\frac{15}{16}$$

Or,

$$4) \underline{11\frac{3}{4}} \quad \underline{2\frac{15}{16}}$$

EXPLANATION.—The mixed number may be reduced to an improper fraction, and the division performed as before.

Or, the number may be divided without being reduced to an improper fraction. Thus, 4 is contained in $11\frac{3}{4}$ 2 times, and a remainder of $3\frac{3}{4}$, or $1\frac{3}{4}$; and $1\frac{3}{4}$ divided by 4 equals $\frac{1}{4}$. Hence the result is $2\frac{15}{16}$.

Divide :

16. $17\frac{3}{8}$ by 7.

19. $25\frac{5}{8}$ by 8.

22. $125\frac{4}{7}$ by 7.

17. $15\frac{1}{4}$ by 4.

20. $31\frac{5}{8}$ by 6.

23. $153\frac{3}{8}$ by 8.

18. $18\frac{6}{7}$ by 5.

21. $46\frac{4}{7}$ by 9.

24. $200\frac{3}{4}$ by 9.

25. The cost of 5 tickets from Troy, N.Y., to Bethlehem, N.H., was \$40 $\frac{1}{2}$. How much did each cost?

26. The rain interfered with a laborer's working $3\frac{1}{2}$ working days in 2 weeks. How many days did he work on an average per week?

27. A farmer forgot the price which he received for his butter per pound, but he knew that he received \$2 $\frac{2}{3}$ for 8 pounds. At that rate, what did he receive per pound?

28. An estate of \$3956 $\frac{2}{3}$ was divided equally among 5 heirs. What sum did each receive?

29. The earnings of 5 men for 10 days were \$87 $\frac{1}{2}$. What were the average earnings of each per day?

30. If a man can reap a field in $10\frac{1}{4}$ days, in what time can 12 men reap it?

31. If a locomotive runs $142\frac{8}{15}$ miles in 6 hours, what is the average rate of speed per hour?

32. A field containing $125\frac{1}{2}$ square rods was divided into 10 lots of equal size. How large were the lots?

184. To divide an integer by a fraction.

1. How many tenths of a dollar are there in a dollar?

2. At $\$ \frac{1}{10}$ each, how many quires of paper can be bought for \$1? For \$2? For \$3? For \$4?

3. When butter is worth $\$ \frac{1}{4}$ per pound, how many pounds can be bought for \$1? For \$2? For \$3?

4. How many penknives, at $\$ \frac{1}{2}$ apiece, can be bought for \$1? For \$2? How many at $\$ \frac{2}{3}$ apiece can be bought for \$2? For \$4?

5. If a boy spends $\frac{1}{3}$ of an hour in going to school, in how many days will he spend 3 hours? 6 hours? 9 hours?

6. A man gave his daughter $\$ \frac{3}{4}$ a month to spend as she pleased. In how many months would he give her \$9?

7. The retail price of one kind of molasses is $\$ \frac{2}{3}$ per gallon. How many gallons can be bought for \$8?

8. A young man spends $\frac{1}{2}$ of each day in work and recreation. In what time will he spend 12 days in this way?

9. How many times is $\frac{1}{2}$ contained in 5? In 9? In 12?

WRITTEN EXERCISES.

185. 1. Divide 5 by $\frac{5}{7}$, or find how many times $\frac{5}{7}$ is contained in 5.

$$5 + \frac{3}{7} = \frac{5 \times 7}{3} = 11\frac{2}{7}.$$

Or,
 $5 = 5\frac{5}{7}; \frac{5}{7} + \frac{5}{7} = 11\frac{2}{7}.$

EXPLANATION. — Since $\frac{1}{2}$ is contained in 1 $\frac{1}{2}$ times, it is contained in 5, 5 times $\frac{1}{2}$ times, or 35 times, and $\frac{5}{7}$ is contained in 5 $\frac{1}{2}$ of 35 times, or $11\frac{2}{7}$ times. Or, since in 5 there are $5\frac{5}{7}$, $\frac{5}{7}$ is contained in $5\frac{5}{7}$, or $11\frac{2}{7}$ times.

1. Cancellation can often be used advantageously.

2. Mixed numbers may be reduced to improper fractions.

Find the quotients of :

- | | | | |
|----------------------------|---------------------------|---------------------------|----------------------------|
| 2. $18 \div \frac{3}{4}$. | 7. $47 + \frac{3}{4}$. | 12. $68 + \frac{1}{8}$. | 17. $225 + \frac{2}{5}$. |
| 3. $21 \div \frac{4}{3}$. | 8. $38 + \frac{9}{10}$. | 13. $71 + \frac{1}{8}$. | 18. $387 + \frac{5}{8}$. |
| 4. $16 \div \frac{3}{8}$. | 9. $21 + \frac{5}{7}$. | 14. $43 + \frac{9}{10}$. | 19. $477 + \frac{5}{27}$. |
| 5. $24 \div \frac{5}{6}$. | 10. $36 + \frac{5}{11}$. | 15. $29 + \frac{1}{6}$. | 20. $642 + \frac{1}{21}$. |
| 6. $36 \div \frac{3}{8}$. | 11. $43 + \frac{8}{15}$. | 16. $35 + \frac{1}{7}$. | 21. $936 + \frac{1}{17}$. |

Divide the following :

- | | | |
|-----------------------------------|----------------------------|----------------------------|
| 22. 29 by $4\frac{1}{4}$. | 24. 36 by $8\frac{1}{4}$. | 26. 33 by $7\frac{1}{4}$. |
| 23. 32 by $3\frac{1}{2}$. | 25. 91 by $5\frac{1}{3}$. | 27. 54 by $6\frac{1}{3}$. |
| 28. Divide 24 by $5\frac{1}{4}$. | | |

$5\frac{1}{4}) 24$

$\overline{7 \quad 7}$

$39) 168 (4\frac{1}{4}$

EXPLANATION. — By changing the dividend and divisor to 7ths, the division is performed as in simple division.

Divide the following :

- | | | |
|----------------------------|----------------------------|-----------------------------|
| 29. 36 by $7\frac{3}{4}$. | 32. 59 by $4\frac{1}{4}$. | 35. 39 by $7\frac{1}{4}$. |
| 30. 46 by $3\frac{5}{6}$. | 33. 36 by $3\frac{5}{6}$. | 36. 35 by $16\frac{1}{2}$. |
| 31. 52 by $6\frac{1}{2}$. | 34. 96 by $4\frac{7}{8}$. | 37. 46 by $8\frac{3}{4}$. |

38. When cloth is $\$.10\frac{1}{2}$ per yard, how many yards can be bought for $\$ 3.36$?

39. My lot has a frontage on the street of 385 feet. How many rods is it in front, $16\frac{1}{2}$ feet being a rod ?

40. A cheap kind of flour sells for $\$ 3\frac{1}{2}$ per barrel. How many barrels can be bought for $\$ 76\frac{7}{8}$?

41. A farmer bought a wagon for $\$ 68$, paying for it in apples at $\$ 1\frac{1}{2}$ per barrel. How many barrels did he give for it ?

42. Boots are commonly sold in cases containing 12 pairs. If a dealer paid $\$ 99$ for a quantity of boots at $\$ 2\frac{1}{4}$ a pair, how many cases did he purchase ?

186. To divide a fraction by a fraction.

- At $\$ \frac{1}{2}$ each, how many books can be bought for $\$ \frac{5}{8}$?
At $\$ \frac{2}{3}$?
- Among how many children can $\frac{2}{3}$ of a melon be divided, if each child is given $\frac{1}{6}$? $\frac{2}{3}$? $\frac{3}{4}$?
- If cream is worth $\$ \frac{1}{10}$ per pint, how many pints can be bought for $\$ \frac{6}{5}$? If $\$ \frac{4}{15}$ per pint? If $\$ \frac{4}{10}$?
- How many pitchers, each holding $\frac{2}{3}$ of a gallon, can be filled from $\frac{1}{2}$ of a gallon?
- If a man uses $\frac{1}{16}$ of a pound of butter each day, how many days will $\frac{1}{8}$ of a pound last him?
- How many times is $\frac{1}{2}$ contained in $\frac{1}{2}$? $\frac{1}{2}$ in $\frac{8}{7}$?
 $\frac{1}{2}$ in $\frac{9}{7}$? $\frac{1}{2}$ in $\frac{1}{2}$ (or 1)? $\frac{1}{2}$ in 1? $\frac{1}{2}$ in 1? $\frac{1}{2}$ in 1? $\frac{1}{2}$ in $\frac{9}{7}$? $\frac{1}{2}$ in $\frac{9}{7}$? $\frac{1}{2}$ in $\frac{8}{7}$?
- If a train moves $\frac{2}{3}$ of a mile each minute, how many minutes will it require to go $3\frac{1}{2}$ miles? How many thirds are there in $3\frac{1}{2}$ miles?
- If it takes $\frac{1}{3}$ of a yard of lace to trim a bonnet, how many bonnets can be trimmed with $2\frac{1}{2}$ yards of lace?
- How many aprons can be made from $4\frac{1}{2}$ yards of muslin, if each apron requires $1\frac{1}{2}$ yards?
How many half yards are there in $1\frac{1}{2}$ yards?
How many half yards are there in $3\frac{1}{2}$ yards?
- How many times are 3 half yards contained in 6 half yards?
- How many books can be bought with $\$ 6\frac{3}{4}$, at $\$ 1\frac{1}{2}$ apiece?
- How many pitchers, each holding $2\frac{3}{4}$ pints, will be required to hold $13\frac{3}{4}$ pints of milk?
- Among how many children can $\frac{1}{2}$ of a pound of candy be divided, if each child is given $\frac{1}{8}$ of a pound?
- How many eighths are there in $\frac{1}{2}$?
- If a coat requires $\frac{3}{4}$ of a dozen buttons, how many coats will require $4\frac{1}{2}$ dozen buttons?

186. With \$ $\frac{3}{4}$, how many yards of cloth can be bought at \$ $\frac{1}{2}$ per yard?

WRITTEN EXERCISES.

187. 1. Divide $\frac{7}{8}$ by $\frac{2}{3}$, or find how many times $\frac{2}{3}$ is contained in $\frac{7}{8}$.

$\frac{7}{8} \div \frac{2}{3} = \frac{7}{8} \times \frac{3}{2} = \frac{21}{16}$. EXPLANATION.—Since it is required to find how many times $\frac{2}{3}$ is contained in $\frac{7}{8}$, we first

find how many times $\frac{1}{2}$ is contained in 1, and then find $\frac{3}{2}$ of that result.

Since $\frac{1}{2}$ is contained in 1 3 times, $\frac{2}{3}$ is contained in 1 one half of 3 times, or $\frac{3}{2}$ times.

Since $\frac{2}{3}$ is contained in 1 $\frac{1}{2}$ times, in $\frac{7}{8}$ it will be contained $\frac{7}{8}$ of $\frac{3}{2}$ times, or $\frac{21}{16}$ times.

188. From the explanation given, it is apparent that:

A fraction may be divided by a fraction by multiplying the dividend by the divisor inverted.

1. Use cancellation whenever it is possible to do so.
2. Reduce mixed numbers and integers to improper fractions.

2. Divide $\frac{8}{5}$ by $\frac{2}{3}$.

SOLUTION.— $\frac{8}{5} \div \frac{2}{3} = \frac{8}{5} \times \frac{3}{2} = \frac{24}{10}$. Or, $\frac{8}{5} = \frac{16}{10}$, and $\frac{2}{3} = \frac{20}{10}$. Therefore, $\frac{16}{10} + \frac{20}{10} = \frac{36}{10}$.

189. Hence it is seen that:

Another method of dividing a fraction by a fraction is to reduce the fractions to similar fractions, and then to divide the numerator of the dividend by the numerator of the divisor.

Divide:

- | | | |
|---------------------------------------|--|---|
| 3. $\frac{7}{8}$ by $\frac{6}{11}$. | 9. $8\frac{1}{2}$ by $\frac{4}{5}$. | 15. $26\frac{1}{4}$ by $4\frac{3}{4}$. |
| 4. $1\frac{1}{2}$ by $\frac{7}{9}$. | 10. $3\frac{7}{12}$ by $1\frac{1}{2}$. | 16. $19\frac{1}{4}$ by $3\frac{3}{4}$. |
| 5. $\frac{8}{9}$ by $\frac{2}{7}$. | 11. $9\frac{9}{14}$ by $2\frac{2}{3}$. | 17. $36\frac{1}{4}$ by $\frac{9}{10}$. |
| 6. $\frac{5}{12}$ by $\frac{2}{15}$. | 12. $7\frac{6}{11}$ by $4\frac{1}{2}$. | 18. $18\frac{1}{4}$ by $\frac{2}{3}$. |
| 7. $\frac{9}{10}$ by $\frac{7}{8}$. | 13. $5\frac{3}{4}$ by $\frac{9}{7}$. | 19. $16\frac{4}{5}$ by $3\frac{3}{4}$. |
| 8. $1\frac{1}{8}$ by $\frac{9}{20}$. | 14. $12\frac{5}{8}$ by $7\frac{5}{12}$. | 20. $7\frac{1}{4}$ by $1\frac{1}{8}$. |

21. How many times may $2\frac{1}{2}$ gallons be drawn from a barrel that holds $31\frac{1}{2}$ gallons?
22. How many bushels of potatoes, at $\$1\frac{1}{2}$ a bushel, may be bought with $\$5$?
23. How many barrels of apples, each holding $2\frac{1}{2}$ bushels, can be filled with 18 bushels?
24. At $\$1\frac{1}{2}$ a basket for peaches, how many baskets of peaches shall I get for $\$8$?
25. When flour is $\$7\frac{1}{2}$ per barrel, how many barrels of flour can be bought for $\$59$?
26. How many pounds of beef, at $11\frac{1}{4}$ per pound, can be bought for $87\frac{1}{2}$?
27. How many books, at $\$3\frac{1}{2}$ per volume, can be purchased for $\$31\frac{1}{2}$?
28. When oranges are $12\frac{1}{2}$ per dozen, how many dozen can be bought for $98\frac{1}{2}$?
29. How many plows, at $\$11\frac{1}{2}$ each, can be bought for $\$67\frac{1}{2}$?
30. How many acres of land, at $\$43\frac{1}{2}$ per acre, can be bought for $\$237\frac{1}{2}$?
31. A railroad $16\frac{1}{2}$ miles long cost $\$66,937$. What was the cost per mile?
32. If $4\frac{1}{2}$ hogsheads hold $25\frac{1}{2}$ bushels, what will 1 hogshead hold?
33. If a field containing $54\frac{1}{2}$ acres yielded 1165 bushels of rye, what was the average crop per acre?
34. Divide $\frac{3}{4}$ of $\frac{5}{6}$ of $\frac{7}{8}$ of $\frac{5}{7}$ by $\frac{2}{3}$ of $\frac{8}{5}$ of $\frac{3}{2}$.

$$\left(\frac{2}{3} \times \frac{3}{4} \times \frac{5}{7} \times \frac{7}{9}\right) \div \left(\frac{6}{5} \times \frac{2}{3} \times \frac{5}{8}\right) = \frac{2}{3} \times \frac{3}{4} \times \frac{5}{7} \times \frac{7}{9} \times \frac{5}{6} \times \frac{8}{2} \times \frac{3}{5} = \frac{5}{9}$$

EXPLANATION. — In solving examples like this, *all the factors of the divisor should be inverted*. All the integers and mixed numbers should be changed to improper fractions before the factors of the divisor are inverted. Use cancellation to abridge the process.

Divide:

35. $\frac{2}{3}$ of $\frac{4}{5}$ of $\frac{3}{4}$ of $\frac{6}{15}$ by $\frac{2}{3}$ of $\frac{4}{5}$ of $\frac{1}{11}$ of $\frac{2}{3}$ of 6.
36. $\frac{2}{3}$ of $\frac{4}{5}$ of $\frac{3}{4}$ of $\frac{7}{15}$ by $\frac{2}{3}$ of $\frac{4}{5}$ of $\frac{2}{3}$ of 8.
37. $\frac{2}{3}$ of $\frac{3}{7}$ of $\frac{8}{5}$ of $\frac{7}{12}$ by $\frac{1}{2}$ of $\frac{2}{3}$ of $\frac{7}{12}$ of 24.
38. $\frac{7}{10}$ of $\frac{2}{3}$ of $\frac{2}{3}$ of $\frac{1}{2}$ by $\frac{1}{2}$ of $\frac{2}{3}$ of $\frac{1}{2}$ of 36.
39. $\frac{2}{3}$ of $\frac{4}{5}$ of $\frac{1}{15}$ of $\frac{8}{5}$ by $\frac{2}{3}$ of $\frac{1}{11}$ of $\frac{2}{3}$ of $\frac{4}{5}$ of 30.
40. $\frac{1}{2}$ of $\frac{2}{3}$ of $\frac{8}{5}$ of $\frac{8}{10}$ by $\frac{2}{3}$ of $\frac{1}{11}$ of $\frac{8}{10}$ of $\frac{1}{2}$ of 44.
41. $\frac{2}{3}$ of $\frac{4}{5}$ of $\frac{1}{9}$ of $\frac{2}{3}$ by $\frac{1}{2}$ of $\frac{2}{11}$ of $\frac{2}{3}$ of $\frac{8}{10}$ of 25.
42. $\frac{2}{3}$ of $\frac{4}{5}$ of $\frac{1}{2}$ of $\frac{1}{25}$ by $\frac{2}{3}$ of $\frac{1}{10}$ of $\frac{2}{3}$ of $\frac{2}{3}$ of $\frac{1}{2}$ of 16.
43. $\frac{2}{3}$ of $\frac{1}{10}$ of $\frac{2}{3}$ of $\frac{3}{5}$ by $\frac{2}{3}$ of $\frac{1}{10}$ of $\frac{2}{3}$ of $\frac{8}{12}$ of 10.
44. $\frac{1}{2}$ of $\frac{2}{3}$ of $\frac{2}{3}$ of $\frac{2}{3}$ by $\frac{2}{3}$ of $\frac{2}{3}$ of $\frac{1}{2}$ of $\frac{2}{3}$ of $\frac{1}{2}$ of 20.
45. $\frac{2}{3}$ of $\frac{4}{5}$ of $\frac{1}{7}$ of $\frac{1}{5}$ by $\frac{2}{3}$ of $\frac{1}{2}$ of $\frac{2}{3}$ of $\frac{1}{11}$ of $\frac{5}{11}$ of 45.
46. $\frac{7}{10}$ of $\frac{2}{3}$ of $\frac{2}{3}$ of $\frac{2}{3}$ by $\frac{2}{3}$ of $\frac{1}{2}$ of $\frac{2}{3}$ of $\frac{9}{15}$ of $\frac{1}{11}$ of 18.

190. Expressions of unexecuted division of fractions are frequently written in the form of fractions. Such expressions are sometimes called **Complex Fractions**.

Thus, $\frac{2}{3} + \frac{2}{5}$ may be written $\frac{\frac{2}{3}}{\frac{2}{5}}$; $6\frac{2}{3} + \frac{2}{5}$, $\frac{6\frac{2}{3}}{\frac{2}{5}}$.

1. Find the value of the fractional form, $\frac{\frac{2}{3}}{\frac{2}{11}}$.

SOLUTION.— $\frac{\frac{2}{3}}{\frac{2}{11}} = \frac{2}{3} + \frac{1}{11} = \frac{22}{33} + \frac{3}{33} = \frac{25}{33} = 1\frac{2}{33}$.

Find the value of the following:

2. $\frac{\frac{2}{3}}{\frac{2}{3}}$.
5. $\frac{\frac{5}{3}}{\frac{1}{4}}$.
8. $\frac{6\frac{1}{4}}{24}$.
11. $\frac{7\frac{1}{3}}{6\frac{1}{3}}$.
3. $\frac{\frac{4}{3}}{\frac{2}{3}}$.
6. $\frac{4\frac{1}{2}}{\frac{2}{3}}$.
9. $\frac{1\frac{1}{4}}{1\frac{1}{2}}$.
12. $\frac{\frac{2}{3} \text{ of } 8}{5\frac{1}{2}}$.
4. $\frac{\frac{2}{3}}{\frac{2}{3}}$.
7. $\frac{5}{\frac{2}{3}}$.
10. $\frac{5\frac{1}{3}}{3\frac{1}{4}}$.
13. $\frac{2\frac{1}{6}}{\frac{1}{2} \text{ of } \frac{2}{3}}$.

191. To find the relation of one number to another.

1. There were 5 oranges on a plate. Willie took 1 orange, and Eddie took 2. What part of all the oranges did Willie take? Eddie?

2. What part of 8¢ is 1¢? 3¢? 5¢?

7 is $\frac{7}{10}$ of 10, because it is 7 of the 10 equal parts into which 10 may be divided. What part of 10 is 9?

3. What part of 10 acres are 5 acres? Of 9 quarts are 3 quarts? Of 8 pounds are 2 pounds? Of 10 inches are 2 inches?

4. 1 is what part of 12? 2 of 12? 3 of 12? 8 of 12?

5. 5 melons are what part of 7 melons?

6. \$5 are what part of \$8?

7. 2 fifths are what part of 3 fifths?

8. 3 eighths are what part of 7 eighths?

9. $\frac{2}{3}$ is what part of $\frac{1}{2}$? $\frac{3}{5}$ of $\frac{1}{6}$? $\frac{5}{8}$ of $\frac{11}{12}$?

10. What part of 2 ounces is $\frac{1}{4}$ of an ounce?

SUGGESTION.—2 ounces = $\frac{1}{4}$ ounces. What part of $\frac{1}{4}$ is $\frac{1}{4}$?

11. What part of \$5 are \$2 $\frac{1}{2}$?

12. What part of $\frac{1}{2}$ of a gallon is $\frac{1}{3}$ of a gallon?

SUGGESTION.— $\frac{1}{2} = \frac{3}{6}$; $\frac{1}{3} = \frac{2}{6}$. What part of $\frac{3}{6}$ is $\frac{2}{6}$?

13. What part of 10 quarts are $3\frac{1}{2}$ quarts?

14. What part of 7 acres are $1\frac{1}{4}$ acres?

15. What part of $\frac{1}{4}$ of a rod is $\frac{1}{2}$ of a rod?

16. What part of $2\frac{3}{4}$ pounds are $1\frac{1}{4}$ pounds?

17. What part of 10 is $\frac{1}{5}$? Of 10 is $\frac{2}{5}$? Of 10 is $1\frac{1}{2}$?

WRITTEN EXERCISES.

192. 1. What part of \$100 are \$72?

$\frac{72}{100} = \frac{18}{25}$. EXPLANATION.—\$1 is $\frac{1}{100}$ of \$100; consequently, \$72 are $\frac{18}{25}$ of \$100, or $\frac{18}{25}$ of \$100.

1. When one or both numbers are fractions not having a common denominator, the numbers should be reduced to equivalent fractions having a common denominator.

2. Mixed numbers should be reduced to improper fractions.

What part of:

2. 85 is 30? 7. 25 is $2\frac{1}{2}$? 12. $\frac{1}{2}$ is $\frac{2}{3}$?
 3. 175 is 28? 8. 40 is $3\frac{1}{2}$? 13. $\frac{1}{2}$ is $\frac{1}{10}$?
 4. 12 is $\frac{2}{3}$? 9. 64 is $4\frac{1}{2}$? 14. $3\frac{1}{2}$ is $1\frac{1}{2}$?
 5. 20 is $\frac{1}{2}$? 10. 33 is $1\frac{1}{3}$? 15. $7\frac{1}{2}$ is $4\frac{1}{2}$?
 6. 32 is $\frac{1}{2}$? 11. 72 is $2\frac{1}{2}$? 16. $5\frac{1}{8}$ is $3\frac{1}{4}$?
 17. In a farm of 120 acres there were $35\frac{1}{4}$ acres of pasture land. What part of the farm was used for pasture?
 18. What part of $272\frac{1}{4}$ square feet are 9 square feet?
 19. If a boy can do a piece of work in 25 days, what part of the work can he do in $7\frac{1}{2}$ days?
 20. Miss Reed paid \$ $20\frac{1}{4}$ for a gown, and \$ $6\frac{1}{2}$ for a hat. What part of the price of the dress did the hat cost?
 21. If the working day is 10 hours long, how much will a man who earns \$3 a day get for 6 hours' work?

ALIQUOT PARTS.

193. An aliquot part of a number is any integer or mixed number that will exactly divide it.

ALIQUOT AND OTHER PARTS OF A DOLLAR.

50% = \$ $\frac{1}{2}$.	10% = \$ $\frac{1}{10}$.	$66\frac{2}{3}\%$ = \$ $\frac{2}{3}$.
$33\frac{1}{3}\%$ = \$ $\frac{1}{3}$.	$8\frac{1}{2}\%$ = \$ $\frac{1}{12}$.	75% = \$ $\frac{3}{4}$.
25% = \$ $\frac{1}{4}$.	$6\frac{1}{4}\%$ = \$ $\frac{1}{16}$.	40% = \$ $\frac{2}{5}$.
20% = \$ $\frac{1}{5}$.	5% = \$ $\frac{1}{20}$.	$83\frac{1}{3}\%$ = \$ $\frac{5}{6}$.
$16\frac{2}{3}\%$ = \$ $\frac{1}{6}$.	4% = \$ $\frac{1}{25}$.	$37\frac{1}{2}\%$ = \$ $\frac{3}{8}$.
$12\frac{1}{2}\%$ = \$ $\frac{1}{8}$.	2% = \$ $\frac{1}{50}$.	$87\frac{1}{2}\%$ = \$ $\frac{7}{8}$.

194. 1. What will 12 pounds of butter cost at $33\frac{1}{3}\%$ per pound?

SOLUTION. — $33\frac{1}{3}\% = \frac{1}{3}$. Since 1 pound costs \$ $\frac{1}{3}$, 12 pounds will cost 12 times \$ $\frac{1}{3}$, which is \$ $4\frac{1}{2}$, or \$4.

2. What will 20 readers cost at 25¢ apiece? At 50¢?
3. What will 25 sponges cost at 20¢ apiece?
4. What will 36 yards of ribbon cost at 16½¢ a yard?
5. What will 48 yards of lace cost at 12½¢ a yard?

What is the cost of:

6. 120 pounds of sugar at 5¢? At 8½¢? At 10¢?
7. 12 neckties at 75¢? At 66½¢? At 83½¢?
8. 40 baskets at 37½¢? At 40¢? At 30¢?
9. 80 gallons of molasses at 37½¢? At 62½¢? At 87½¢?
10. 28 dozen pencils at 12½¢? At 16½¢? At 20¢?
11. 16 pairs of gloves at 66½¢? At 83½¢? At 75¢?
12. 24 days' work at \$1.25? At \$1.33½? At \$1.16½?
13. At 16½¢ per pound, how many pounds of raisins can be bought with \$2?

SOLUTION.— $16\frac{1}{2}\text{¢} = \$\frac{1}{8}$. Since 1 pound costs $\$ \frac{1}{8}$, as many pounds can be bought with \$2 as $\$ \frac{1}{8}$ is contained times in \$2, or $\$ \frac{1}{8} \times 8$, which is 12 times. Therefore, 12 pounds of raisins can be bought with \$2.

14. At $33\frac{1}{3}\text{¢}$ a dozen, how many dozen eggs can I buy with \$5? \$3? \$7? \$4?
15. At $37\frac{1}{2}\text{¢}$ apiece, how many books can be bought for $\$2\frac{1}{2}$? $\$3\frac{1}{2}$? $\$4\frac{1}{2}$? $\$5\frac{1}{2}$?
16. For $\$2\frac{1}{2}$, how many yards of muslin can be bought at 10¢ a yard? At $12\frac{1}{2}\text{¢}$? At $16\frac{1}{2}\text{¢}$?

195. To find the whole when a part and the relation of the part to the whole are given.

1. If \$5 is $\frac{1}{4}$ of my money, how much money have I? 10 cents is $\frac{1}{4}$ of my brother's money. How much has he?
2. A man sold 5 gallons of vinegar, which was $\frac{1}{4}$ of all he had. How many gallons had he?
3. A farmer sold 10 quarts of milk per day, which was $\frac{1}{4}$ of what the cows produced. How much did they produce per day?

4. A boy paid \$6 for a coat, and that sum was $\frac{1}{4}$ of his wages for a month. What were his monthly wages?

5. A lad purchased a box of cartridges, fired off 10 of them, and had $\frac{2}{5}$ of them left. How many cartridges were there in the box?

6. A boy borrowed 50 feet of kite string, which was $\frac{1}{10}$ of what another boy had. How many feet of string did the other boy have?

7. A farmer estimated that a hail storm had caused him a loss of 5 bushels of oats per acre, or $\frac{1}{16}$ of his crop. How much did he think his land would yield him per acre?

8. A boy and his sister played backgammon for some time, his sister beating him 15 games, which was $\frac{1}{3}$ of the number of games played. How many did they play?

9. If $\frac{2}{3}$ of a number is 6, how much is $\frac{1}{2}$ of it?

10. If $\frac{3}{4}$ of a number is 9, how much is $\frac{1}{4}$ of it?

11. If $\frac{5}{7}$ of a number is 10, how much is $\frac{1}{7}$ of it?
Since $\frac{1}{7}$ of the number is 2, what is the number?

12. If $\frac{2}{5}$ of a number is 8, how much is $\frac{1}{5}$ of the number?
Since $\frac{1}{5}$ of the number is 4, what is the number?

WRITTEN EXERCISES.

196. 1. 320 is $\frac{1}{8}$ of what number?

$$\frac{1}{8} = 320.$$

EXPLANATION.—Since 320 is $\frac{1}{8}$ of a certain number, $\frac{1}{8}$ of 320, or 64, is $\frac{1}{8}$ of the number; and

$$\frac{1}{8} = 64.$$

since 64 is $\frac{1}{8}$ of the number, the number must be 8 times 64, or 512. Hence 320 is $\frac{1}{8}$ of 512.

$$\text{Number} = 512.$$

Find the number of which

2. 40 is $\frac{5}{8}$.

3. 60 is $\frac{3}{5}$.

4. 85 is $\frac{5}{16}$.

5. 75 is $\frac{3}{4}$.

6. 65 is $\frac{5}{8}$.

7. 81 is $\frac{9}{10}$.

8. 125 is $\frac{5}{8}$.

9. 340 is $\frac{17}{5}$.

10. 560 is $\frac{14}{5}$.

11. 300 is $\frac{15}{7}$.

12. 240 is $\frac{16}{5}$.

13. 330 is $\frac{11}{3}$.

14. $\frac{5}{8}$ is $\frac{1}{4}$.

15. $\frac{7}{20}$ is $\frac{1}{5}$.

16. $\frac{14}{5}$ is $\frac{7}{5}$.

17. $\frac{25}{7}$ is $\frac{5}{7}$.

18. $\frac{85}{7}$ is $\frac{5}{7}$.

19. $\frac{45}{8}$ is $\frac{9}{16}$.

20. A boy sold his bicycle at second hand for $\frac{2}{3}$ of the cost, receiving for it \$80. How much did it cost him?

21. Our tennis court is just $\frac{5}{6}$ of what it was originally, and it is now 120 feet. How long was it originally?

22. The distance between Rochester and Syracuse, N.Y., is 81 miles, and it lacks 4 miles of being $\frac{1}{2}$ of the distance from New York to Buffalo. What is the distance from New York to Buffalo?

23. The width of a river near its source was 45 feet, and that distance was $\frac{3}{10}$ of what it was 20 miles farther down. How wide was it at the latter point?

24. A gardener set out some cabbage plants, but only 80 of them lived, which was just $\frac{5}{7}$ of the number set out. How many did he set out?

25. A way train stopping at all stations averaged 25 miles per hour; but the rate of the train was only $\frac{5}{11}$ of that of a fast express train. How many miles per hour did the express train run?

REVIEW EXERCISES.

ORAL EXERCISES.

197. 1. Change to improper fractions: $3\frac{2}{5}$; $5\frac{3}{7}$; $6\frac{2}{3}$; $5\frac{4}{5}$; $7\frac{1}{4}$.

2. Change to mixed numbers: $2\frac{7}{9}$; $1\frac{8}{5}$; $4\frac{1}{7}$; $5\frac{2}{3}$; $1\frac{9}{4}$; $4\frac{2}{5}$; $5\frac{7}{9}$.

3. Find the sum of $3\frac{1}{2}$, $4\frac{3}{4}$, $5\frac{1}{4}$, $6\frac{1}{2}$, $7\frac{3}{4}$, and $3\frac{3}{4}$.

4. Find the sum of $6\frac{1}{2}$, $4\frac{1}{2}$, $2\frac{3}{4}$, $3\frac{5}{6}$, $4\frac{1}{3}$, and $2\frac{2}{3}$.

5. A man had 3 lots containing respectively $3\frac{1}{2}$ acres, $2\frac{1}{4}$ acres, and $3\frac{1}{8}$ acres. How many acres did they all contain?

6. The expense of a carpet was \$ $12\frac{3}{4}$, and it cost \$ $3\frac{1}{2}$ to put it down. What was the entire cost of the carpet laid in the room?

7. Terrence's uncle gave him \$10 on Christmas, but he spent $\$3\frac{1}{2}$ of it. How much had he left?
8. A father allowed his son \$40 per year for spending money. At the end of the year the boy had $\$18\frac{1}{2}$. How much did he spend?
9. The expense of a Columbia bicycle was \$140; but a boy bought one that pleased him just as well for $\frac{2}{3}$ of that price. How much did it cost him?
10. The price of California peaches was $\$2\frac{1}{4}$ per dozen in July. How much did 8 dozen cost?
11. A man had a field containing $4\frac{1}{2}$ acres, which he divided into lots containing $\frac{1}{7}$ of an acre. How many lots were there?
12. A wagon which cost $\$50\frac{1}{2}$ was sold for $\$46\frac{3}{4}$. How much was the loss?
13. After purchasing a silver watch for $\$19\frac{1}{4}$, I had $\$21\frac{1}{2}$ left. How much had I at first?
14. A man owed for groceries $\$18\frac{1}{2}$; for meat, $\$10\frac{1}{2}$, and for clothes $\$8\frac{1}{2}$. If he paid the debts out of \$50, which he has, how much did he have left?
15. A boy went on a visit to his cousins with \$35 to pay his expenses. On returning he found that he had spent $\frac{2}{3}$ of his money. How much had he left?
16. The proprietor of a boarding house, after buying provisions for the day, found that he had $\$35\frac{1}{4}$ left, which was just $\frac{1}{3}$ of the money he had before making the purchases. How much did he spend?
17. An American flag was purchased by some school children for $\$11\frac{1}{2}$, and the flag-staff cost $\$6\frac{1}{4}$. What did both cost?
18. A man sold a horse for \$160 and a cow for $\frac{2}{3}$ as much. How much did he get for both?
19. A cow was sold for \$60, which was $\frac{1}{3}$ of what a horse was sold for. How much was received for both?

20. After spending $\frac{1}{4}$ of my money, I had \$35 left. How much had I ?
21. A horse traveled $38\frac{1}{2}$ miles in 7 hours. What was his average rate of speed per hour ?
22. A man sold a cow for \$40 and spent $\frac{1}{4}$ of the sum. Afterwards he bought a horse and paid in cash what he had left from the sale of the cow. What did the horse cost him, if the cash payment was $\frac{1}{4}$ of the cost of the horse ?
23. A farm was sold in 2 parts. The first part contained 50 acres and that was $\frac{1}{4}$ of the second part. How many acres were there in the farm ?
24. I spent $\frac{1}{4}$ of my money for books, $\frac{1}{4}$ of it for furniture, and had \$100 left. How much had I at first ?
25. James had a sum of money such that $\frac{1}{2}$ of it was just $\frac{1}{4}$ of Henry's. What part of Henry's was the whole of James's ?
26. One half of a mother's age was $\frac{1}{2}$ of her daughter's. What part of the mother's age was the daughter's ?
27. What part of the daughter's age was the mother's ?
28. $\frac{5}{6}$ multiplied by another fraction gives a product of $\frac{5}{6}$. What is the multiplier ?
29. The difference between $\frac{1}{2}$ and $\frac{1}{4}$ of a number is 8. What is the number ?

WRITTEN EXERCISES.

- 193.** Find the value of :

- $3\frac{1}{2} + 3\frac{1}{2} + 6\frac{1}{2} - 3\frac{1}{2} - \frac{1}{2} - 5\frac{5}{6}$.
- $5\frac{1}{2} + 6\frac{1}{2} + 3\frac{7}{10} - 6\frac{1}{2} - 5\frac{1}{15} + 2\frac{3}{25}$.
- $7\frac{1}{2} + 5\frac{1}{2} - 9\frac{7}{12} + 8\frac{1}{2} - \frac{1}{12} + 5\frac{1}{10}$.
- $(7\frac{1}{2} + 3\frac{1}{2}) \times 5\frac{1}{2} + (3\frac{1}{2} + 1\frac{1}{2}) \times 3\frac{1}{2}$.
- $(3\frac{1}{2} - 1\frac{1}{2}) + 1\frac{1}{2} + (2\frac{1}{2} - 1\frac{1}{2}) \times 6\frac{1}{2} - 5$.
- $(4\frac{1}{2} + 6\frac{1}{2} - 3\frac{1}{2}) + \frac{2}{10} + (3\frac{1}{2} - 2\frac{1}{2}) \times 6$.
- $(4\frac{1}{2} - 3\frac{1}{2} + 7\frac{1}{10} + \frac{1}{2} + 4) + \frac{1}{10} + 8$.
- $(6\frac{1}{2} + 4\frac{1}{2}) \times (5\frac{1}{2} - 3\frac{1}{2}) + (2\frac{1}{2} - 1\frac{1}{2})$.

9. How much will 5 masons earn in $4\frac{1}{2}$ days, at $\$3\frac{1}{2}$ per day?
10. After selling $\frac{1}{3}$ of his load of potatoes to one person, $\frac{1}{4}$ to another, and 25 bushels to another, a farmer had sold his entire load. How many bushels did he sell?
11. One fifth of the expenses of an excursion were paid by Mr. A., $\frac{1}{3}$ of them by Mr. B., and the rest, which was $\$17$, by the rest of the party. What were the expenses of the excursion?
12. A man forgot how many bushels of rye he had sold, but he remembered that he had received $\$185$ for the rye at $\$7$ per bushel. How many bushels did he sell?
13. A man loaned some money, receiving at the end of the year $\$245$ interest for it. If the rate of interest was $\frac{8}{5}\%$ of the sum loaned, how much did he loan?
14. A man finds that he must save $\$600$ per year. He takes boarders, and can accommodate 10. Suppose that he pays $\$2520$ for provisions, servants, and rent, and his rooms are full all the year; how much must he charge per week to save $\$600$?
15. A man of means spent $\frac{1}{3}$ of his life traveling in America, $\frac{1}{4}$ of it in Europe, and the rest of it at his own home. What part of his life did he spend at his own home?
16. If he spent 30 years at his own home, how old was he?
17. If A can do a piece of work in 6 days, what part of it can he do in 1 day? If B can do the same piece of work in 7 days, what part of it can he do in 1 day? If both work together, what part of it can they do in 1 day?
18. If both, working together, could only do $\frac{1}{12}$ of it in 1 day, how many days would they need to complete it? Since both together can do $\frac{1}{12}$ in 1 day, what part of 42 days will they need to complete it?

19. A man bequeathed $\frac{1}{4}$ of his property to his brother, $\frac{1}{2}$ of it to his sister, and the rest, which was \$62500, to a college. How much did he bequeath in all?
20. A can do a piece of work in 3 days, and B can do it in 5 days. In what time can both together do it?
21. A can do a piece of work in 8 days, and B can do it in 10 days. In what time can both do it together?
22. It takes A and B 8 days to do a piece of work which B can do alone in 12 days. In what time can A do it?
23. Two men during a year earned together \$1085; but one of them only earned $\frac{1}{3}$ as much as the other. How much did each earn?
24. Two brothers earn respectively \$2 and \$3 per day. In what time can both together earn \$13?
25. A man who owned $\frac{2}{3}$ of a sash factory sold $\frac{2}{3}$ of his share for \$24000. How much was the factory worth at that rate?
26. Divide $\frac{2}{3}$ of $\frac{4}{5}$ of $4\frac{1}{2}$ by $\frac{3}{5}$ of $\frac{6}{7}$ of $1\frac{1}{2}$.
27. Divide $\frac{4}{5}$ of $\frac{2}{3}$ of $\frac{5}{6}$ of $6\frac{2}{3}$ by $\frac{2}{3}$ of $\frac{3}{4}$ of $2\frac{1}{2}$.
28. Find the sum and the difference of $4\frac{1}{2}$ and $3\frac{1}{2}$; divide the sum by the difference; the difference by the sum; and then find the product of the quotients.
29. Multiply $3\frac{1}{2}$ by $2\frac{1}{2}$; divide $3\frac{1}{2}$ by $2\frac{1}{2}$; and find the difference between the two results.
30. A man spent $\frac{1}{3}$ of his money and \$25 more, and then had \$225 left. How much money had he?
31. A man after spending \$2250 in repairing his house, found that it cost him, including repairs, just $\frac{4}{3}$ of the original cost. What was the original cost?
32. The estimated cost of a building was $\frac{4}{3}$ of its actual cost. If $\frac{1}{3}$ of the estimated expense was \$5000, what was the actual cost of the building?
33. A school in one of our cities had the names of 1039 pupils on its rolls. If there had been one girl more, there

would have been $2\frac{1}{4}$ times as many girls as boys in the school. How many boys and how many girls were enrolled?

34. A man engaged in manufacturing made a profit of $\frac{5}{100}$ on his capital, and this sum, added to his capital, made \$27060. How much capital had he invested in the business?

35. The product of three numbers is $72\frac{1}{4}$. Two of them are $5\frac{3}{4}$ and $9\frac{1}{2}$. What is the other number?

36. If a man walks $8\frac{1}{2}$ miles in $2\frac{1}{2}$ hours, how far can he walk in $5\frac{1}{2}$ hours?

37. A meter, the French unit of length, is very nearly $39\frac{1}{4}$ inches. What is the difference in length between 1 yard and 1 meter? What is the difference between $5\frac{7}{8}$ yards and $5\frac{1}{4}$ meters?

38. My annual income from money loaned at \$6 on \$100 is \$36.60. How much have I loaned?

39. A company owned a cotton factory worth \$350,000. If A and B own $\frac{1}{4}$ of it, what is the value of their share?

40. Find the difference between $40\frac{1}{2}$ and $25\frac{1}{4}$; divide the difference by the sum and determine the quotient.

41. If 6 is added to both terms of the fraction $\frac{2}{5}$, is its value increased or diminished, and how much?

42. A merchant sold $\frac{1}{5}$, $\frac{1}{4}$, and $\frac{1}{3}$ of his tea and had 130 pounds left. How much tea had he?

43. The cost of a live turkey weighing 14 pounds was \$2. If the waste in dressing was $\frac{1}{4}$ of the weight of the turkey, for how much per pound must I sell it to gain \$1 on the cost?

44. One half of my goods was destroyed by fire; $\frac{1}{2}$ the remainder was destroyed by water and the rest cost me \$8650. What was the entire cost of the goods?

45. A carpenter alone can build a house in 25 days, but with the assistance of his son he can build it in 15 days. In how many days can the son build it?

DECIMAL FRACTIONS.

199. 1. If anything is divided into 10 equal parts, what is each part called?

2. If $\frac{1}{10}$ of a line is divided into 10 equal parts, what part of the whole line is each part? How much is $\frac{1}{10}$ of $\frac{1}{10}$?

3. If $\frac{1}{100}$ of a line is divided into 10 equal parts, what part of the line is each part? How much is $\frac{1}{10}$ of $\frac{1}{100}$?

4. How many hundredths are equal to $\frac{1}{10}$?

5. How many thousandths are equal to $\frac{1}{100}$?

6. The divisions of anything into tenths, hundredths, thousandths, etc., are called *decimal divisions*.

200. One or more of the decimal divisions of a unit is called a **Decimal Fraction**.

The word *decimal* is derived from the Latin word *decem*, which means ten.

Decimal fractions are usually called *decimals*.

201. Since decimals have the same law of increase and decrease as integers, the *denominator* of the fraction may be indicated by the *position* of the figures.

202. The figures in the *first place* at the right of units represent *tenths*; in the *second*, *hundredths*; in the *third*, *thousandths*; in the *fourth*, *ten-thousandths*; etc.

203. A period called the **Decimal Point** is placed before the decimal.

Thus, 9 represents $\frac{9}{10}$.48 represents $\frac{48}{100}$.059 represents $\frac{59}{1000}$
.2 represents $\frac{2}{10}$.08 represents $\frac{8}{100}$.007 represents $\frac{7}{1000}$

.5 represents $\frac{5}{10}$.04 represents $\frac{4}{100}$.006 represents $\frac{6}{1000}$
 .8 represents $\frac{8}{10}$ 1.00 represents $1\frac{00}{100}$.029 represents $\frac{29}{1000}$
 1.1 represents $1\frac{1}{10}$ 1.12 represents $1\frac{12}{100}$ 5.005 represents $5\frac{5}{1000}$

NUMERATION TABLE.

Hundreds.	Tens.	Units.	Decimal point.	Tenths.	Hundredths.	Thousands.	Ten-thousandths.	Hundred-thousandths.	Millionths.
7	9	6		5	8	4	6	9	4
<u>INTEGERS.</u>			<u>DECIMALS.</u>						

The orders below millionths in their order are: ten-millionths, hundred-millionths, billionths, ten-billionths, hundred-billionths, etc.

204. What order of decimals occupies

- | | | |
|-------------|-------------|-------------|
| 1st place ? | 2d place ? | 7th place ? |
| 3d place ? | 4th place ? | 2d place ? |
| 5th place ? | 6th place ? | 4th place ? |

What decimal place is occupied by

- | | | |
|---------------|-----------------------|------------------|
| Tenths ? | Ten-thousandths ? | Ten-millionths ? |
| Thousandths ? | Millionths ? | Hundredths ? |
| Hundredths ? | Hundred-thousandths ? | Thousands ? |

EXERCISES IN NUMERATION.

205. 1. Read 42.356.

EXPLANATION. — The decimal part of the number expresses 3 tenths, 5 hundredths, 6 thousandths, or 356 thousandths. The whole expression is, therefore, read: 42 and 356 thousandths.

206. Hence it is evident that :

In reading a decimal, the decimal should be read as an integer, and the denomination of the right-hand figure should be added.

In reading an integer and decimal, use the word *and* only between the integral and decimal parts of the number.

Read the following:

2. .31	9. 2.63	16. 217.305
3. .73	10. 3.10	17. 301.1031
4. .281	11. 5.031	18. 130.2005
5. .032	12. 7.003	19. 800.80061
6. .0578	13. 72.070	20. 256.0032
7. .5601	14. 26.303	21. 250.00071
8. .5013	15. 31.120	22. 504.03062

EXERCISES IN NOTATION.

207. 1. Express decimaly 54 thousandths.

EXPLANATION. — Since 54 thousandths is equal to 5 hundredths and 4 thousandths, 4 is written in the thousandths' place, 5 in hundredths' place, and since there are no tenths, 0 in tenths' place. A decimal point is then placed before the tenths. Hence 54 thousandths = .054.

208. Hence it is evident that:

In expressing a fraction decimaly, the numerator of the fraction is written and ciphers are prefixed, if necessary, to indicate the denominator. The decimal point is then placed before tenths.

Express the following as decimals:

2. Six tenths. Four tenths. Nine tenths.
3. Thirteen hundredths. Thirty hundredths.
4. Fifteen thousandths. Two hundred one thousandths.
5. Seventeen thousandths. Six ten-thousandths.
6. Fourteen and three hundred thirty-two hundred thousandths. Five millionths.
7. Fifty-one and seven hundred forty-two millionths.
8. Fifty and seventy-five ten-thousandths.
9. Thirty-one and sixty-four hundred-thousandths.
10. Seventy-nine and nine ten-millionths.
11. Three hundred and three millionths.
12. Sixty and sixty-seven millionths.
13. 312 millionths. 201 ten-millionths.

14. 705 ten-millionths. 605 and 703 billions.
 15. 631 ten-billionths. 25 and 25 ten-millionths.
 16. $\frac{230}{1000}$. 19. $\frac{341}{1000}$. 22. $23710\frac{8}{100}$.
 17. $\frac{277}{1000}$. 20. $210\frac{227}{1000}$. 23. $6751\frac{9}{1000}$.
 18. $6\frac{6}{1000}$. 21. $601\frac{51}{1000}$. 24. $3002\frac{250}{1000}$.

REDUCTION OF DECIMALS.

209. To reduce dissimilar decimals to similar decimals.

1. How do the fractions $\frac{6}{10}$, $\frac{60}{100}$, and $\frac{600}{1000}$ compare in value? How do the decimals expressing them compare in form? What is the effect of annexing a cipher to a decimal?
 2. How do the fractions $\frac{6}{10}$, $\frac{6}{100}$, and $\frac{6}{1000}$ compare in value? How do the decimals expressing them compare in form? What is the effect of prefixing a cipher to a decimal?
 3. How does the number of places in a decimal compare with the number of ciphers in the denominator?

210. Hence it is evident that:

1. *Annexing ciphers to a decimal does not alter its value.*
 2. *Each decimal cipher prefixed to a decimal diminishes the value of the decimal tenfold.*
 3. *The number of places in a decimal is equal to the number of ciphers in the denominator of its equivalent common fraction.*

WRITTEN EXERCISES.

211. 1. Reduce .6, .46, and .095 to similar fractions.

EXPLANATION. — Since the lowest order of the given decimals is thousandths, all the decimals must be changed to thousandths. Since annexing ciphers to a decimal does not alter its value, we give to each number three decimal places by annexing ciphers, thus making them similar by reducing them all to thousandths.

2. Reduce to similar decimals: .0321, .00623, .00025, .75.
3. Reduce to ten-thousandths: .68, .0785, .063, .251, .27.
4. Express 8 tenths as hundredths; as thousandths.
5. Change to similar decimals: .003, .02, .2100, .6530.

212. To reduce a decimal to a common fraction.

WRITTEN EXERCISES.

1. Reduce .75 to its equivalent common fraction.

EXPLANATION. — .75 expressed as a common fraction is $\frac{75}{100} = \frac{3}{4}$, which, being reduced to its smallest terms, is $\frac{3}{4}$.

213. From the explanation it is apparent that:

A decimal may be reduced to a common fraction by omitting the decimal point and supplying the proper denominator.

1. The result should be expressed in its lowest terms.

Reduce the following to common fractions in their smallest terms :

- | | | | |
|---------|----------|-------------|-------------|
| 2. .75. | 6. .125. | 10. .0045. | 14. .00375. |
| 3. .50. | 7. .875. | 11. .075. | 15. .00625. |
| 4. .35. | 8. .625. | 12. .00875. | 16. .05375. |
| 5. .80. | 9. .375. | 13. .0250. | 17. .0875. |

214. To reduce a common fraction to a decimal.

1. How many tenths are there in $\frac{1}{2}$? In $\frac{1}{3}$?
2. How many hundredths are there in $\frac{1}{2}$? In $\frac{1}{3}$? In $\frac{1}{4}$?
3. How many hundredths are there in 1? In $\frac{1}{5}$? In $\frac{1}{6}$?
4. How many thousandths are there in 1? In $\frac{1}{5}$? In $\frac{1}{6}$?

WRITTEN EXERCISES.

1. Reduce $\frac{5}{6}$ to an equivalent decimal.

EXPLANATION. — $\frac{5}{6}$ is $\frac{1}{2}$ of 5, or of 50 tenths; and $\frac{1}{2}$ of 50 tenths is 6 tenths, and 2 tenths remaining. 2 tenths are equal to 20 hundredths, and $\frac{1}{2}$ of 20 hundredths is 2 hundredths and 4 hundredths remaining. 4 hundredths are equal to 40 thousandths, and $\frac{1}{2}$ of 40 thousandths is 5 thousandths. Hence $\frac{5}{6}$ is equal to 6 tenths + 2 hundredths + 5 thousandths, or .625.

216 From the process it is evident that:

A common fraction may be reduced to a decimal by annexing ciphers to the numerator and dividing by the denominator, and pointing off as many decimal places in the quotient as there are ciphers annexed.

In many cases the division is not exact. In such instances the remainder may be expressed as a common fraction, or the sign + may be employed after the decimal to show that the result is not complete; thus $\frac{1}{3} = .166\bar{6}$, or $.166+$.

Reduce the following to decimals:

- | | | | | | |
|---------------------|---------------------|----------------------|-----------------------|-----------------------|-----------------------|
| 2. $\frac{8}{10}$. | 6. $\frac{8}{5}$. | 10. $\frac{1}{10}$. | 14. $\frac{3}{4}$. | 18. $\frac{1}{4}$. | 22. $\frac{4}{5}$. |
| 3. $\frac{5}{4}$. | 7. $\frac{3}{20}$. | 11. $\frac{1}{11}$. | 15. $\frac{1}{15}$. | 19. $\frac{8}{3}$. | 23. $\frac{5}{12}$. |
| 4. $\frac{3}{8}$. | 8. $\frac{1}{40}$. | 12. $\frac{1}{11}$. | 16. $\frac{1}{625}$. | 20. $3\frac{1}{4}$. | 24. $\frac{1}{80}$. |
| 5. $\frac{1}{20}$. | 9. $\frac{2}{3}$. | 13. $\frac{4}{7}$. | 17. $3\frac{1}{4}$. | 21. $6\frac{1}{30}$. | 25. $\frac{3}{100}$. |

ADDITION OF DECIMALS.

- 217.** 1. What is the sum of $\frac{8}{10}$ and $\frac{8}{5}$? Of .4 and .7?
 2. What is the sum of $\frac{1}{10}$ and $\frac{1}{100}$? Of .08 and .09?
 3. What is the sum of $\frac{1}{4}$ and $\frac{1}{10}$? Of .5 and .25?

WRITTEN EXERCISES.

- 218.** 1. What is the sum of 2.85, 1.625, and 3.0235?

2.8500 or 2.85 **EXPLANATION.**—The numbers are written so that units stand under units, tenths under tenths, etc. The decimals may be made similar by annexing ciphers and then added, care being taken to place the decimal point in the sum directly under the decimal point in the numbers added. Or, since only units of one order are found in any column, it is not necessary, in practice, to make the decimals similar.

Find the sum of the following:

- | | |
|------------------------------|------------------------------|
| 2. $8.75 + .369 + 75.212$. | 4. $3.801 + 2.875 + 1.625$. |
| 3. $1.89 + 62.625 + 3.939$. | 5. $1.25 + 4.275 + 48.35$. |

6. $.356 + 1.275 + 32.0325$. 11. $3.495 + 1.87 + 3.25$.
 7. $15.815 + 17.21 + 31.41$. 12. $38.835 + 8.9764 + 1.385$.
 8. $.2735 + 1.596 + 3.25$. 13. $1.8035 + 3.00412 + 31.025$.
 9. $2.948 + 3.7642 + 85.035$. 14. $62.589 + 3.875 + 1.3759$.
 10. $46.2 + 381.25 + 2.985$. 15. $28.48 + 17.578 + 34.752$.

16. What is the sum of 8 tenths, 12 hundredths, 75 thousandths, 69 ten-thousandths, 21 hundredths?

17. What is the sum of 312 ten-thousandths, 8 millions, 65 thousandths, 413 ten-millionths, 9 tenths?

18. What is the sum of 82 hundredths, 510 thousandths, 6 tenths, 80 ten-thousandths?

19. What is the sum of 303 thousandths, 4108 millions, 635 ten-thousandths, 803 ten-millionths?

20. A bicyclist traveled on his wheel 51.875 miles in one day, 45.75 the next, and 35.375 the next day. How far did he travel during these three days?

21. A ticket agent sold in one month tickets to the value of \$18,756.75, in the next \$17,983.25, in another \$2001.875. How much were his sales during those three months?

SUBTRACTION OF DECIMALS.

219. 1. From $\frac{5}{10}$ take $\frac{5}{10}$. From .18 take .13.
 2. From $\frac{7}{100}$ take $\frac{5}{100}$. From .035 take .026.
 3. From 4 take $\frac{5}{10}$. From 8 take .8.

WRITTEN EXERCISES.

220. 1. From 46.57 subtract 9.46325.

$$46.57000 = 46.57$$

$$9.46325 = \underline{9.46325}$$

$$37.10675 \quad 37.10675$$

EXPLANATION. — The numbers are written so that units stand under units, tenths under tenths, etc. The decimals may be made similar and then subtracted, care being taken to place the decimal point in the remainder directly under the decimal point in the number subtracted. The ciphers may be supposed to be annexed when we subtract, and consequently need not be written.

	2.	3.	4.	5.	6.
From	25.38	39.85	87.35	46.81	24.37
Take	<u>14.05</u>	<u>29.755</u>	<u>59.355</u>	<u>21.35</u>	<u>12.358</u>
	7.	8.	9.	10.	11.
From	72.189	78.895	26.825	34.24	21.12
Take	<u>35.976</u>	<u>53.5987</u>	<u>21.3452</u>	<u>30.16</u>	<u>12.31</u>

12. What is the difference between 6.325 and 1.0345?
13. What is the difference between 6.45 and 2.3375?
14. From 65 hundredths subtract 65 thousandths.
15. From 162 thousandths subtract 261 millionths.
16. Find the value of $23.2 - .0325 + 6.95 - 3.4876$.
17. Find the value of $6.594 - 2.0341 - .70325 + 8.963$.
18. Find the value of $81.8 + 35.625 - 38.875 - 2.0034$.
19. Find the value of $4.6 + 2.635 - .8925 - .0359$.
20. From a cistern that contained 30.5 barrels of water, 25.75 barrels were drawn off. How much water remains in the cistern?
21. A man bought a tub of butter for \$35.75, paying \$10 in cash and the rest in groceries. How much did he pay in groceries?
22. A man whose income was \$10,000 spent in one year \$7375.87. How much did he save that year?

MULTIPLICATION OF DECIMALS.

- 221.** 1. What is the product of $\frac{1}{10} \times \frac{1}{10}$? Of $.1 \times .1$? Of $.5 \times .5$? How does the number of ciphers in the denominator of the product of these fractions compare with the number of ciphers in the denominators of the factors?
2. What is the product of $\frac{1}{10} \times \frac{1}{100}$? Of $.1 \times .01$? Of $.5 \times .05$? How does the number of ciphers in the denominator of the product of these fractions compare with the number of ciphers in the denominators of the factors?

3. How does the number of decimal places in a decimal fraction compare with the number of ciphers in its denominator?

4. How, then, does the number of decimal places in the product of two or more decimals compare with the number of places in the factors?

222. It is apparent, therefore, that:

The number of decimal places in the product of two or more decimals is equal to the number of decimal places in the factors.

WRITTEN EXERCISES.

223. 1. What is the product of .275 multiplied by .17?

$$\begin{array}{r} .275 \\ \times .17 \\ \hline \end{array}$$
 EXPLANATION. — Since 275 and 17 are the numerators of the fractions, in multiplying the fractions, we must multiply 275 by 17, which gives 4675 for a new numerator.

$$\begin{array}{r} 1925 \\ \times 275 \\ \hline .04675 \end{array}$$
 Since thousandths multiplied by hundredths produce hundred-thousandths, the product of the fractions is 4675 hundred-thousandths, or .04675.

Or the number of decimal places in the product will be five, the number of decimal places in both factors. Art. 222.

224. Hence, it is evident that:

To multiply decimals we multiply as if the numbers were integers, and from the right of the product point off as many figures for decimals as there are decimal places in both factors.

If the product does not contain a sufficient number of decimal places, the deficiency must be supplied by *prefixing* ciphers.

Multiply:

- | | | |
|----------------|-----------------|-------------------|
| 2. .25 by .75. | 5. .75 by 2.06. | 8. .876 by .375. |
| 3. .24 by 3.4. | 6. .93 by .047. | 9. .433 by 2.75. |
| 4. .57 by 68. | 7. .39 by .875. | 10. 72.2 by .055. |

11. 3.62 by .0037. 15. 2.37 by 7.563. 19. .1895 by .0436.
 12. 1.98 by 2.034. 16. 2.53 by .00635. 20. 6.81 by 0.0875.
 13. 25.7 by 3.887. 17. .3147 by .0052. 21. 578 by .0934.
 14. 15.8 by .0855. 18. .1963 by .00638. 22. 0.765 by .067.

23. Multiply 5.372 by 100.

EXPLANATION. — Since each removal of a figure one place to the left increases its value tenfold, the removal of the decimal point one place to the right multiplies by 10, and its removal two places to the right multiplies by 100; or, the process may be performed in the ordinary manner.

24. Multiply 27.65 by 10. 28. Multiply 6.8735 by 100.
 25. Multiply 49.675 by 10. 29. Multiply .3768 by 1000.
 26. Multiply 2.795 by 100. 30. Multiply 6.835 by 100.
 27. Multiply 8.935 by 100. 31. Multiply .7634 by 1000.
 32. Multiply 89.92 by 10. By 100. By 1000. By 10,000.
 33. Multiply 68.3025 by 100. By 1000. By 10,000.
 34. Which is larger and how much, the product of .625 and .375, or their sum?

35. What will 52.75 bushels of oats cost at \$.45 per bushel?

36. A merchant sold 24.25 yards of muslin at \$.125 a yard. How much did he receive for it?

37. A man bought 214 bags of barley. What did they cost him, if each bag held 2.5 bushels, and the price was \$.96 a bushel?

38. A tailor bought 4.5 yards of cassimere at \$2.25 a yard, and 6.25 yards of broadcloth at \$6.50 a yard. How much more did the broadcloth cost than the cassimere?

39. How many acres are left in a farm of 563.27 acres, if 3 fields of 57.135 acres each are sold from it?

40. A steamship in crossing the Atlantic sailed at an average speed of 325.75 miles per day. If another steamer sailed from the same port at the same time at the rate of 395.35 miles per day, how far were they apart in 5.75 days?

DIVISION OF DECIMALS.

- 225.** 1. When a number is divided by 3, what part of it is found? By 6? By 8? By 15?
 2. How much is $\frac{1}{3}$ of 9? $9 \div 3$? $\frac{1}{6}$ of 15? $15 \div 5$?
 3. How much is $\frac{1}{3}$ of 9 tenths? $\frac{1}{6}$ of .8? $\frac{1}{6}$ of .6? $\frac{1}{6}$ of .36?
 4. How much is $\frac{1}{3}$ of 25 hundredths? $\frac{1}{6}$ of .24? $\frac{1}{6}$ of .36?

WRITTEN EXERCISES.

- 226.** 1. Find 1 sixth of 109.92, or divide 109.92 by 6.

6) 109.92 EXPLANATION.—One sixth of 10 tens is 1 ten and 4 tens remaining; 1 sixth of 4 tens remaining and 9 units, or 49 units, is 8 units and 1 unit remaining; 1 sixth of 1 unit remaining and 9 tenths, or 19 tenths, is 3 tenths and 1 tenth remaining; 1 sixth of 1 tenth remaining and 2 hundredths, or 12 hundredths, is 2 hundredths. Therefore, one sixth of 109.92 is 18.32, or the quotient of $109.92 \div 6$ is 18.32.

2. What is 1 fifth of 86.4055? 5. $\$234.54 \div 9 = ?$
 3. What is 1 eighth of 94076.8? 6. $\$907.34 \div 7 = ?$
 4. What is 1 eleventh of 1086.415? 7. $\$324.784 \div 8 = ?$
 8. Find 1 eighth of 9675 acres.

8) 9675.000 EXPLANATION.—We divide as in simple division 1209.375 until we come to the remainder, 3 acres. This we change to 30 tenths and continue the division thus: 1 eighth of 30 tenths is 3 tenths and 6 tenths, or 60 hundredths, remainder. 1 eighth of 60 hundredths is 7 hundredths and 4 hundredths, or 40 thousandths, remainder; 1 eighth of 40 thousandths is 5 thousandths. Therefore, 1 eighth of 9675 acres is 1209.375 acres.

How much is:

9. $\frac{1}{4}$ of $\$23.50$? 12. $\frac{1}{4}$ of 392.25? 15. $8.643 \div 6$?
 10. $\frac{1}{5}$ of $\$36.75$? 13. $\frac{1}{5}$ of 468.36? 16. $85.86 \div 9$?
 11. $\frac{1}{6}$ of $\$59.64$? 14. $\frac{1}{6}$ of 693.34? 17. $3.845 \div 5$?
 18. A man received $\$3278$ for 5 acres of land. What was the price per acre?
 19. An estate valued at $\$35,267$ was divided equally among 8 children. What was each child's share?

227. The number of decimal places in the quotient.

1. What is the product of $.5 \times .7$? How many decimal places does it contain? If .35 is divided by .7, what will the quotient be? How many decimal places does it contain?
2. What is the product of $.9 \times .05$? If .045, the product, is divided by .9, one of the factors, what is the other factor? How many decimal places does it contain?
3. Since the product contains as many decimal places as there are in all the factors, if the product of two factors is given and one of the factors, how may the number of decimal places in the other factor be found?
4. Since the dividend may be regarded as the product of two factors, one of which is the divisor, and the other the quotient, how may the number of decimal places in the quotient be found from those in the dividend and divisor?

228. It is therefore evident that:

The quotient will contain as many decimal places as the number of decimal places in the dividend exceeds the number in the divisor.

WRITTEN EXERCISES.

229. 1. Divide .15652 by .043.

.043).15652(3.64

$$\begin{array}{r}
 129 \\
 \underline{275} \\
 258 \\
 \underline{172} \\
 172
 \end{array}$$

EXPLANATION. — Since the dividend is equal to the product of the divisor and quotient, if 15652, the numerator of the dividend, is divided by 43, the numerator of the divisor, the result obtained will be the numerator of the quotient. Dividing, it is found to be 364. Hence, the numerator of the quotient is found precisely as in simple division. The denominator of the quotient may be found by dividing the denominator of the dividend by the denominator of the divisor, that is by dividing 100000 by 1000, which gives 100. Therefore, the quotient is $\frac{364}{100}$, or 3.64. Or,

The quotient will contain as many decimal places as the number of decimal places in the dividend exceeds the number in the divisor.
Art. 228.

230. Hence it is evident that:

In dividing decimals, we divide as if the numbers were integers, and from the right of the quotient point off as many places for decimals as the number of decimal places in the dividend exceeds the number in the divisor.

1. If the quotient does not contain a sufficient number of decimal places, the deficiency must be supplied by *prefixing* ciphers.
2. Before commencing the division, the number of decimal places in the dividend *should be made at least equal* to the number of decimal places in the divisor.
3. When there is a remainder after using all the figures of the dividend, annex decimal ciphers and continue the division.
4. For business purposes, it is not necessary to carry the division further than to four or five decimal figures in the quotient.

Divide:

- | | |
|------------------------|----------------------|
| 2. 78.12 by 3.6. | 17. 14.91 by 70. |
| 3. 12.32 by 3.12. | 18. 8.25 by 1.5. |
| 4. 1675.8 by .49. | 19. .00864 by .24. |
| 5. 11.904 by 3.1 | 20. 107.5 by .043. |
| 6. 452.92 by .052. | 21. .0009 by .125. |
| 7. 63.168 by .94. | 22. .07245 by .23. |
| 8. .15275 by .325. | 23. 800 by .0004. |
| 9. .12376 by .0364. | 24. 12.126 by .235. |
| 10. .0184428 by .5123. | 25. 167.544 by 71.6. |
| 11. .29184 by .0057. | 26. 56.112 by 0.56. |
| 12. 4.5872 by .0122. | 27. 2.1828 by .034. |
| 13. .056448 by 15.68. | 28. 4066.2 by .648. |
| 14. .0816785 by 6.235. | 29. .08 by 1.611. |
| 15. .0983253 by .0673. | 30. 114.87 by .0035. |
| 16. 5.97066 by 35.73. | 31. 30000 by .00003. |

32. Divide 325.47 by 100.

100)325.47 EXPLANATION. — Since each removal of a figure one place to the right decreases its value tenfold, the removal of the decimal point one place to the left divides by 10, and the removal two places by 100.

231. Hence it is evident that :

To divide by 1 with any number of ciphers annexed, the decimal point should be removed as many places to the left as there are ciphers annexed to the 1.

- | | |
|---|----------------------------|
| 33. Divide 392.5 by 100. | 36. Divide 4.825 by 100. |
| 34. Divide 26.45 by 10. | 37. Divide 38.62 by 1000. |
| 35. Divide 369.5 by 1000. | 38. Divide 42.85 by 10000. |
| 39. How many yards of cloth, at \$ 5.75 per yard, can be bought for \$ 153.525 ? | |
| 40. How many barrels of apples, at \$ 1.25 per barrel, can be bought for \$ 18.43 $\frac{1}{4}$? | |
| 41. How many cords of wood, at \$ 5.62 $\frac{1}{2}$ per cord, can be bought for \$ 1495.12 $\frac{1}{2}$? | |

ACCOUNTS AND BILLS. .

232. A record of the debts and credits between two parties is called an **Account**.

233. A statement of the quantity and price of the articles sold, and the value of the whole, is called a **Bill**.

A bill is *receipted* when the words *Received Payment*, or *Paid*, are written at the bottom, and the seller's name is signed either by himself or by some authorized person.

234. The following abbreviations are in common use :

@,	At.	Do.,	The same.	Mdse.,	Merchandise.
%,	Account.	Doz.,	Dozen.	No.,	Number.
Acc't,	Account.	Dr.,	Debtor.	Pay't,	Payment.
Bal.,	Balance.	Fr't,	Freight.	Pd.,	Paid.
Bbl.,	Barrel.	Hhd.,	Hogshead.	Per,	By.
Bo't,	Bought.	Inst.,	This month.	Rec'd,	Received.
Co.,	Company.	Int.,	Interest.	Yd.,	Yard.
Cr.,	Creditor.	Lb.,	Pound.	Yr.,	Year.

1.

Brooklyn, N.Y., June 25, 1892.

Mr. Horace E. Gray,

Bought of JONES & PURDY.

4 bags Coffee,	300 lb.	@	\$35 $\frac{1}{2}$	\$106	50
3 chests Tea,	260 "	"	.87 $\frac{1}{2}$	227	50
8 boxes Raisins,	200 "	"	.12 $\frac{1}{2}$	25	00
2 bbl. Coffee-sugar,	432 "	"	.06 $\frac{1}{2}$	27	00
2 bbl. Crackers,	150 "	"	.05 $\frac{1}{2}$	8	25
Received payment,				\$394	25

Jones & Purdy,

Der G.S.

Make out in proper form, find the footings of the following bills and accounts, and receipt them:

2. Miss Lucy Graham bought of Luckey, Platt & Co. 15 yards of calico @ 10 cents; 36 yards of sheeting @ 18 $\frac{1}{4}$ cents; 2 pairs of gloves @ \$1.50; 1 sun-umbrella @ \$ 3.75; 5 yards of Hamburg edging @ 25 cents; 6 handkerchiefs @ 37 $\frac{1}{2}$ cents; and 7 pairs of hose @ 50 cents.

3. Messrs. Henry Davis & Co. bought of James Harkness 150 barrels of flour @ \$ 5.90; 105 bushels of wheat @ \$ 1.20; 325 bushels of corn @ \$.68; 675 bushels of oats @ \$.40; and 50 barrels of potatoes @ \$ 1.25.

4. Mrs. Charles Reid in account with Roger Caldwell: August 7, 1892, 9 yards cashmere @ \$.75; $\frac{1}{4}$ yard velvet @ \$ 1.50; 12 yards lawn @ .12 $\frac{1}{2}$; 1 $\frac{1}{2}$ yards silesia @ \$.30; $\frac{1}{4}$ yards silk at \$ 1.50.

5. Darwin Howard bought of Stone & Bacon 15 pounds nails @ 4 $\frac{1}{4}$ ¢; 4 pairs of hinges @ 37 $\frac{1}{2}$ ¢; 18 dozen screws @ 11¢; 5 dozen milk-pans @ \$ 2.75; 7 locks @ 75¢; 2 saws @ \$ 1.35; 3 hammers @ 85¢; and 2 planes @ \$ 1.50.

REVIEW.

WRITTEN EXERCISES.

235. 1. How much will 2 dozen pairs of kid gloves cost at \$1.25 a pair?

2. A man paid out the following sums: For a pair of horses, \$375; for a carriage, \$295; for harnesses, \$115. How much did he pay for all?

3. A farmer sold 12.35 cords of wood at \$4.75 per cord. How much did he receive for it?

4. A man purchased 8 tons of coal at \$5.75 per ton, and 7 cords of hard wood at \$5.50 per cord. How much did he pay for fuel?

5. If illuminating gas is sold at \$2.50 per thousand cubic feet, for how much gas will \$17.50 pay?

6. Find the sum of 15 dollars 9 cents 3 mills, 12 dollars 5 cents 4 mills, 18 dollars 14 cents 7 mills, and 16 dollars 10 cents.

7. If 5.5 yards of ribbon cost \$2.75, what will 9.75 yards cost?

8. If a clerk earns \$520 per year and spends \$110 for clothes, \$215 for board, and \$75.50 for other expenses, how long will it take him to save \$1000?

9. If 54,600 letters were mailed from a post-office in 30 days, what was the average number mailed per day?

10. How many pairs of curtains can be bought for \$414, if 12 pairs are bought for \$138?

11. A drover bought 280 head of cattle for \$12,740, and sold them at a gain of \$8.75 per head. How much did he receive for them per head?

12. A man paid \$87.50 for materials to paint his house, and he paid 3 painters \$2.50 per day for 4.5 days to do the work. How much was the entire expense of painting his house?

13. The distance between two places is $168\frac{1}{4}$ miles. How long will it take a bicyclist to ride the distance at 12.5 miles per hour?
14. How much will one dozen each of cups and saucers cost, if the cups cost $\$.74\frac{1}{4}$ and the saucers $\$.67\frac{1}{4}$ apiece?
15. What must I pay for 3 tables at $\$12.75$ each, 2 chairs at $\$9.50$ each, 6 chairs at $\$3.75$ each, and one sofa at $\$25.50$?
16. A merchant sold 35.5 yards of cambric at 25 cents per yard and gained $\$1.775$. How much did it cost him per yard?
17. During the Civil War a dollar bill was, at one time, worth only 65 cents in gold. How many dollars in gold were 520 dollars in paper money worth at that time?
18. A farmer received for his barley $\$151.37$, for his oats $\$196.84$, for his corn as much as for barley and oats, for his rye $\$75.93$, and for his wheat as much as for all the others. For how much did he sell his grain?
19. A dealer bought 25 barrels of flour, each containing 196 pounds, for $\$112.50$. How much did it cost him per pound? How much will be his entire gain if he sells it at $3\frac{1}{4}$ cents per pound?
20. A merchant paid the following sums, on the average, for expenses during each month: For rent, $\$275$; for heating and lighting, $\$115$; for clerk hire, $\$375$; and for other expenses, $\$85$. What will be his net gain annually, if his gross profits are $\$12,560$?
21. An ice-dealer put into his ice-house 1315 tons of ice. He sold at different times the following quantities: 216.75 tons, 396.5 tons, 542.7 tons. If the rest melted during the summer, how many tons melted, and how many tons did he sell?
22. If a freight train runs 15.75 miles per hour, in how many hours will it run 189 miles?

DENOMINATE NUMBERS.

236. A number used in connection with some specified thing is called a **Concrete Number**.

Thus, 5 days, 6 men, 4 feet, 8 pounds are concrete numbers.

237. A number used without reference to any particular thing is called an **Abstract Number**.

Thus, 5, 6, 4, 8, are abstract numbers.

238. A concrete number in which the unit of measure is established by law or custom is called a **Denominate Number**.

Thus, 5 gallons, 8 rods, 5 acres, are denominate numbers.

239. A denominate number which is composed of units of one denomination only is called a **Simple Denominate Number**.

Thus, 3 feet, 8 ounces, 5 quarts, are simple denominate numbers.

240. A denominate number which is composed of units of two or more denominations that are related to each other is called a **Compound Denominate Number**.

Thus, 5 yards 2 feet 8 inches is a compound denominate number.

MEASURES OF LENGTH.

241. In measuring length or distance, **Measures of Length** or **Linear Measures** are used.

TABLE.

12 inches (in.)	= 1 foot (ft.).
3 feet	= 1 yard (yd.).
$5\frac{1}{2}$ yards }	= 1 rod (rd.).
$16\frac{1}{2}$ feet	= 1 mile (mi.).

EXERCISES.

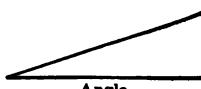
1. How many in. are there in 2 ft.? In 3 ft.? In 4 ft.?
2. How many ft. are there in 4 yd.? In 5 yd.?
3. How many yd. are there in 4 rd.? In 6 rd.? In 8 rd.?
4. How many ft. are there in 2 rd.? In 5 rd.? In 10 rd.?
5. How many rd. are there in 2 mi.? In 3 mi.?
6. How many ft. are there in 24 in.? In 36 in.?
7. How many yd. are there in 9 ft.? In 15 ft.? In 24 ft.?
8. How many rd. are there in 33 ft.? In 66 ft.?
9. How many mi. are there in 640 rd.? In 960 rd.?
10. How many in. are there in $\frac{1}{2}$ ft.? In $\frac{3}{4}$ ft.? In $\frac{5}{8}$ ft.?
11. How many ft. are there in $\frac{1}{3}$ yd.? In $1\frac{1}{2}$ yd.?
12. How many in. are there in 2 ft.? In 2 ft. 6 in.?
13. How many ft. are there in 1 yd.? In 1 yd. 2 ft.? In 2 yd.? In 2 yd. 1 ft.? In 3 yd. 2 ft.?
14. How many in. are there in 1 yd.? In 1 yd. 1 ft.?

MEASURES OF SURFACE.

242. Anything that has only length and breadth is called a **Surface**.

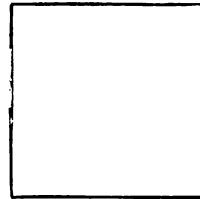
Thus, this page, the side of a block, etc., are surfaces.

243. The difference in the direction of two lines that meet is called an **Angle**.



Angle.

244. A figure that has four equal sides and four equal angles is called a **Square**.



Square.

1. A *square inch* is a square each of whose sides is an *inch* long; a *square foot* is a square each of whose sides is a *foot* long.

2. The angles of a square are called *right angles*.

TABLE.

144 square inches (sq. in.)	= 1 square foot (sq. ft.).
9 square feet	= 1 square yard (sq. yd.).
30 $\frac{1}{2}$ square yards }	= 1 square rod (sq. rd.).
272 $\frac{1}{2}$ square feet }	
160 square rods	= 1 acre (A.).
640 acres	= 1 square mile (sq. mi.).

EXERCISES.

1. How many sq. in. are there in 2 sq. ft.? In 3 sq. ft.?
2. How many sq. ft. are there in 3 sq. yd.? In 4 sq. yd.?
3. How many sq. rd. are there in 2 A.? In 3 A.?
4. How many sq. yd. are there in 18 sq. ft.? In 36 sq. ft.?
5. How many sq. ft. are there in 288 sq. in.? In 432 sq. in.?
6. How many A. are there in $\frac{1}{4}$ sq. mi.? In $\frac{1}{4}$ sq. mi.?
7. How many sq. rd. are there in $\frac{1}{2}$ A.? In $\frac{1}{2}$ A.?
8. How many sq. ft. are there in $\frac{1}{8}$ sq. yd.? In $\frac{3}{8}$ sq. yd.?
9. How many sq. in. are there in $\frac{1}{2}$ sq. ft.? In $\frac{1}{2}$ sq. ft.?
10. How many sq. ft. are there in 2 sq. ft. 12 sq. in.?
In 5 sq. ft. 80 sq. in.?

MEASURES OF VOLUME.

245. Anything that has length, breadth, and thickness is called a **Solid**.

246. The number of solid units in a body is called its **volume** or **solid contents**.

247. A solid having six equal square sides, called faces, is a **Cube**.

A solid whose faces are each a square inch is a *cubic inch*; a solid whose faces are each a square foot is a *cubic foot*.

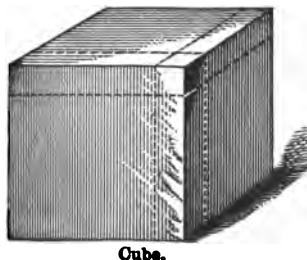


TABLE.

1728 cubic inches (cu. in.)	= 1 cu. ft. (cu. ft.).
27 cubic feet	= 1 cubic yard (cu. yd.).
128 cubic feet	= 1 cord of wood or stone (C.).

EXERCISES.

1. How many cu. in. are there in 2 cu. ft.? In 5 cu. ft.?
2. How many cu. ft. are there in 4 cu. yd.? In 8 cu. yd.?
3. How many cu. ft. are there in 2 C.? In 3 C.?
4. How many cu. yd. are there in 54 cu. ft.? In 81 cu. ft.?
5. How many cu. in. are there in $\frac{1}{2}$ cu. ft.? In $\frac{1}{4}$ cu. ft.?
6. How many cu. ft. are there in $\frac{1}{2}$ C.? In $\frac{1}{8}$ C.?
7. How many cu. ft. are there in 2 cu. yd. 3 cu. ft.?
8. How many cu. in. are there in 1 cu. ft. 72 cu. in.?

LIQUID MEASURES.

248. Measures used in measuring all fluids and liquids are called **Liquid Measures**.

TABLE.

4 gills (gi.)	= 1 pint (pt.).
2 pints	= 1 quart (qt.).
4 quarts	= 1 gallon (gal.).

1. In determining the capacity of cisterns, reservoirs, etc., $31\frac{1}{2}$ gallons are considered a barrel (bbl.), and 2 barrels, or 63 gallons, a hogshead (hhd.).

2. A gallon contains 231 cubic inches.

EXERCISES.

1. How many gi. are there in 2 pt.? In 3 pt.? In 5 pt.?
2. How many pt. are there in 3 qt.? In 5 qt.? In 8 qt.?
3. How many qt. are there in 5 gal.? In 6 gal.?
4. How many gal. are there in 24 qt.? In 40 qt.?
5. How many qt. are there in 16 pt.? In 20 pt.?
6. How many gal. are there in 16 pt.? In 20 pt.?

7. How many pt. are there in $\frac{1}{2}$ gal.? In $\frac{1}{4}$ gal.?
8. How many pt. are there in 3 qt.? In 3 qt. 1 pt.?
9. How many gi. are there in 1 pt. 3 gi.? In 1 pt. 2 gi.?
10. How many pt. are there in 1 gal.? In 2 gal. 3 qt.?

DRY MEASURES.

249. Measures used in measuring fruit, roots, vegetables, and grain are termed **Dry Measures**.

TABLE.

2 pints (pt.)	= 1 (qt.).
8 quarts	= 1 peck (pk.).
4 pecks	= 1 bushel (bu.).

1. A bushel contains 2150.4 cubic inches.
2. A quart and a pint dry measure contain a little more than the same liquid measures.

EXERCISES.

1. How many pt. are there in 5 qt.? In 7 qt.?
2. How many qt. are there in 5 pk.? In 4 pk.?
3. How many pk. are there in 3 bu.? In 6 bu.?
4. How many bu. are there in 24 pk.? In 36 pk.?
5. How many pk. are there in 40 qt.? In 32 qt.?
6. How many qt. are there in 16 pt.? In 20 pt.?
7. How many pk. are there in $\frac{1}{2}$ bu.? In $\frac{1}{4}$ bu.?
8. How many pk. are there in 2 bu. 3 pk.?

AVOIRDUPOIS WEIGHT.

250. In weighing groceries and all heavy articles as hay, grain, coal, including all metals except gold and silver, **Avoirdupois Weight** is used.

TABLE.

16 ounces (oz.)	= 1 pound (lb.).
100 pounds	= 1 hundred-weight (cwt.).
20 hundred-weight	= 1 ton (T.).
2000 pounds	

EXERCISES.

1. How many oz. are there in 2 lb.? In 3 lb.? In $4\frac{1}{2}$ lb.?
2. How many lb. are there in 4 cwt.? In 6 cwt.?
3. How many lb. are there in 3 T.? In 5 T.? In $6\frac{1}{4}$ T.?
4. How many cwt. are there in 300 lb.? In 400 lb.?
5. How many T. are there in 2500 lb.? In 3000 lb.? In 5000 lb.?
6. How many lb. are there in $\frac{1}{2}$ T.? In $\frac{1}{4}$ T.? In $\frac{3}{4}$ T.?
7. How many oz. are there in $\frac{1}{10}$ T.? In $\frac{7}{10}$ T.?

TROY WEIGHT.

- 251.** In weighing gold, silver, and jewels **Troy Weight** is used.

TABLE.

24 grains (gr.) = 1 pennyweight (pwt.).

20 pennyweights = 1 ounce (oz.).

12 ounces = 1 pound (lb.).

The Troy lb. contains 5760 gr.; the Avoirdupois lb., 7000 gr.

The Troy oz. contains 480 gr.; the Avoirdupois oz., $437\frac{1}{2}$ gr.

EXERCISES.

1. How many gr. are there in 2 pwt.? In 4 pwt.?
2. How many pwt. are there in 2 oz.? In 4 oz.? In 6 oz.?
3. How many oz. are there in 4 lb.? In $4\frac{1}{2}$ lb.? In $5\frac{1}{4}$ lb.?
4. How many gr. are there in 1 oz.? In $\frac{1}{2}$ oz.? In $\frac{1}{4}$ oz.?
5. How many lb. are there in 24 oz.? In 36 oz.?
6. How many oz. are there in 40 pwt.? In 50 pwt.?
7. How many pwt. are there in 48 gr.? In 60 gr.?
8. How many oz. are there in 1 lb.? In 2 lb. 4 oz.?

APOTHECARIES' WEIGHT.

252. In weighing medicines for prescriptions Apothecaries' Weight is used by apothecaries and physicians.

TABLE.

20 grains (gr.)	= 1 scruple (sc., or ʒ).
3 scruples	= 1 dram (dr., or ʒ).
8 drams	= 1 ounce (oz., or ʒ).
12 ounces	= 1 pound (lb., or lb.).

EXERCISES.

1. How many gr. are there in 3 sc. ? In 4 sc. ? In $5\frac{1}{2}$ sc. ?
2. How many sc. are there in 4 dr. ? In 5 dr. ? In 10 dr. ?
3. How many dr. are there in 3 oz. ? In 8 oz. ? In $9\frac{1}{2}$ oz. ?
4. How many oz. are there in 2 lb. ? In $4\frac{1}{2}$ lb. ? In $5\frac{1}{2}$ lb. ?
5. How many lb. are there in 48 oz. ? In 60 oz. ? In 66 oz. ?
6. How many oz. are there in 24 dr. ? In 40 dr. ? In 52 dr. ?
7. How many dr. are there in 12 sc. ? In 18 sc. ? In 22 sc. ?
8. How many sc. are there in 40 gr. ? In 50 gr. ? In 90 gr. ?
9. How many dr. are there in 2 oz. 3 dr. ? In 3 oz. 5 dr. ?

DIVISIONS OF TIME.**253.****TABLE.**

60 seconds (sec.)	= 1 minute (min.).
60 minutes	= 1 hour (hr.).
24 hours	= 1 day (da.).
7 days	= 1 week (wk.).
365 days	= 1 year (yr.).
366 days	= 1 leap year (l. yr.).
100 years	= 1 century (C.).

Centennial years whose number is exactly divisible by 400, and other years whose number is exactly divisible by 4, are *Leap Years*.

There are 12 months in a year. The names and the number of days in each are as follows:

- | | |
|---------------------------------|-----------------------------|
| 1. January (Jan.) 31 days. | 7. July (July) 31 da. |
| 2. February (Feb.) 28 or 29 da. | 8. August (Aug.) 31 da. |
| 3. March (Mar.) 31 da. | 9. September (Sept.) 30 da. |
| 4. April (Apr.) 30 da. | 10. October (Oct.) 31 da. |
| 5. May (May) 31 da. | 11. November (Nov.) 30 da. |
| 6. June (June) 30 da. | 12. December (Dec.) 31 da. |

The 29th day of February is the day added to make the leap year.

The following rhyme will serve to fix in memory the days in each month:

Thirty days hath September,
April, June, and November.
All the rest have thirty-one,
Save February, which alone
Hath twenty-eight, and one day more
We add to it one year in four.

EXERCISES.

1. How many sec. are there in 5 min.? In 6 min.? In 10 min.?
2. How many min. are there in 2 hr.? In 3 hr.?
3. How many hr. are there in 2 da.? In 4 da.? In $\frac{1}{2}$ da.?
4. How many da. are there in 3 wk.? In 6 wk.?
5. How many wk. are there in 14 da.? In 35 da.?
6. How many hr. are there in 240 min.? In 180 min.?
7. How many min. are there in 120 sec.? In 300 sec.?
8. How many da. are there in 4 wk. 3 da.? In 5 wk.?
9. Which of the following named years are leap years?
1880? 1892? 1900? 2000? 1864? 1870?
10. How many da. are there in June? In Dec.? In Mar.? In Aug.? In Jan.? In Sept.? In Feb.?

COUNTING.

254. The following denominations are used in counting.

TABLE.

12 things = 1 dozen (doz.).
 12 dozen = 1 gross (gr.).
 12 gross = 1 great gross (G. gr.).
 20 things = 1 score.

STATIONERS' TABLE.

255. The following table is used in the paper trade:

TABLE.

24 sheets = 1 quire (qr.).
 20 quires = 1 ream (R.).
 2 reams = 1 bundle.
 5 bundles = 1 bale.

EXERCISES.

1. How many things are there in 3 doz. ? In 6 doz. ?
2. How many are 3 score ? 3 score and 10 ?
3. How many doz. are there in 3 gr. ? In 1 G. gr. ?
4. How many sheets are there in 2 qr. ? In $5\frac{1}{2}$ qr. ?
5. How many qr. are there in 3 R. ? In 6 R. ? In $6\frac{1}{2}$ R. ?
6. How many qr. are there in 48 sheets ? In 96 sheets ?

256. MISCELLANEOUS EXERCISES.

- | | | | |
|----------------|---------------|----------------------------|-------------|
| 1. 4 bu. | = ? pk. | 11. $1\frac{1}{2}$ cu. ft. | = ? cu. in. |
| 2. 3 ft. | = ? in. | 12. 32 qt. | = ? pk. |
| 3. 5 lb. | = ? oz. (Ap.) | 13. $2\frac{1}{4}$ A. | = ? sq. rd. |
| 4. 10 qt. | = ? pt. | 14. 3 yd. | = ? in. |
| 5. 18 sq. ft. | = ? sq. yd. | 15. 48 qt. | = ? gal. |
| 6. 6 oz. | = ? pwt. | 16. $3\frac{1}{4}$ hr. | = ? min. |
| 7. 70 da. | = ? wk. | 17. 10 R. | = ? qr. |
| 8. 60 units | = ? doz. | 18. 72 gr. | = ? pwt. |
| 9. 3 qr. | = ? sheets | 19. 32 oz. (Av.) | = ? lb. |
| 10. 54 cu. ft. | = ? cu. yd. | 20. 4 score | = ? units. |

REDUCTION OF DENOMINATE NUMBERS.

257. The process of changing a denominate number from one denomination to another without altering its value is called **Reduction**.

258. The process of changing a denominate number to an equivalent number of a *lower* denomination is called **Reduction to Lower Denominations**, or **Reduction Descending**.

259. The process of changing a denominate number to an equivalent number of a higher denomination is called **Reduction to Higher Denominations**, or **Reduction Ascending**.

REDUCTION TO LOWER DENOMINATIONS.

260. 1. How many inches are there in 7 yd. 1 ft. 4 in.?

7

3

EXPLANATION. — Since there are 3 ft. in 1 yd., in 7 yd. there are 7 times 3 ft., or 21 ft., and 21 ft. + 1 ft. = 22 ft. Since there are 12 in. in 1 ft., in 22 ft. there are 22 times 12 in., or 264 in., and 264 in. + 4 in. = 268 in. Therefore 7 yd. 1 ft. 4 in. = 268 in.

21

1

22

12

264

4

268

The business method of reducing denominate numbers to lower denominations is more convenient than that given in the explanation. It is illustrated by the process in the margin.

Reduce to lower denominations :

- | | |
|--------------------------------|-----------------------------------|
| 2. 4 yd. 2 ft. 8 in. | 23. 4 rd. 3 yd. 2 ft. 6 in. |
| 3. 12 yd. 1 ft. 7 in. | 24. 5 rd. 2 yd. 1 ft. 8 in. |
| 4. 15 yd. 2 ft. 10 in. | 25. 8 rd. 1 yd. 2 ft. 10 in. |
| 5. 27 yd. 2 ft. 11 in. | 26. 3 sq.yd. 4 sq. ft. 18 sq. in. |
| 6. 6 gal. 3 qt. 1 pt. | 27. 5 sq.yd. 8 sq. ft. 100 sq.in. |
| 7. 13 gal. 2 qt. 1 pt. | 28. 2 A. 110 sq. rd. 5 sq. yd. |
| 8. 15 gal. 1 qt. 1 pt. 2 gi. | 29. 8 A. 25 sq. rd. 4 sq. yd. |
| 9. 18 gal. 3 qt. 1 pt. 3 gi. | 30. 2 cu.yd. 10 cu.ft. 85 cu.in. |
| 10. 4 hr. 15 min. 15 sec. | 31. 4 cu.yd. 6 cu.ft. 714 cu.in. |
| 11. 10 hr. 12 min. 18 sec. | 32. 3 lb. 2 oz. 7 dr. 1 sc. |
| 12. 18 hr. 20 min. 35 sec. | 33. 4 lb. 8 oz. 5 dr. 2 sc. |
| 13. 4 cwt. 20 lb. 8 oz. | 34. 3 R. 10 qr. 18 sheets. |
| 14. 8 cwt. 75 lb. 12 oz. | 35. 4 R. 8 qr. 10 sheets. |
| 15. 2 T. 5 cwt. 80 lb. 12 oz. | 36. 3 lb. 6 oz. 10 pwt. |
| 16. 4 T. 7 cwt. 35 lb. 9 oz. | 37. 5 lb. 10 oz. 3 sc. |
| 17. 4 lb. 6 oz. 18 pwt. 10 gr. | 38. 6 da. 10 hr. 25 min. |
| 18. 8 lb. 4 oz. 10 pwt. 8 gr. | 39. 8 bu. 3 pk. 7 qt. 1 pt. |
| 19. 5 lb. 5 oz. 15 pwt. 9 gr. | 40. 8 rd. 4 yd. 2 ft. 6 in. |
| 20. 3 bu. 3 pk. 5 qt. 1 pt. | 41. 30 bu. 3 pk. 6 qt. |
| 21. 5 bu. 2 pk. 7 qt. 1 pt. | 42. 5 A. 140 sq. rd. 3 sq. yd. |
| 22. 6 bu. 1 pk. 4 qt. 1 pt. | 43. 7 mi. 100 rd. 13 ft. 6 in. |

44. Reduce .75 yd. to feet and inches.

.75	
3	
2.25	
12	
3.00	

EXPLANATION.—Since in 1 yd. there are 3 ft., in .75 yd. there are .75 of 3 ft. or 2.25 ft. Since in 1 ft. there are 12 in., in .25 ft. there are .25 of 12 in., or 3 in. Therefore, .75 yd. = 2 ft. 3 in.

.75 yd. = 2 ft. 3 in.

- | | | | |
|------------------------|-----------------------|-----------------------|----------------|
| 45. $\frac{3}{4}$ cwt. | 48. $\frac{3}{4}$ yd. | 51. .4 cu. yd. | 54. .73 wk. |
| 46. $\frac{4}{5}$ rd. | 49. .6 bu. | 52. $\frac{4}{5}$ mi. | 55. .3125 gal. |
| 47. $\frac{7}{8}$ wk. | 50. 45 T. | 53. $\frac{4}{5}$ A. | 56. .475 yd. |

REDUCTION TO HIGHER DENOMINATIONS.

262. 1. Reduce 235 pints to bushels, pecks, etc.

2	235 pt.
8	117 qt. 1 pt.
4	14 pk. 5 qt.
	3 bu. 2 pk.

EXPLANATION. — Since there are 2 pt. in 1 qt., in 235 pt. there are as many quarts as 2 pt. are contained times in 235 pt., or 117 qt. 1 pt.

235 pt. = 3 bu. 2 pk. 5 qt. 1 pt. Since there are 8 qt. in 1 pk., there are as many pecks in 117 qt. as 8 qt. are contained times in 117 qt., or 14 pk. 5 qt.

Since there are 4 pk. in 1 bu., in 14 pk. there are as many bushels as 4 pk. are contained times in 14 pk., or 3 bu. 2 pk.

Therefore, 235 pt. are equal to 3 bu. 2 pk. 5 qt. 1 pt.

263. Hence it is evident that:

To reduce a denominate number to units of higher denominations, the given number is divided by the number indicating how many units of the given denomination make one of the next higher denomination.

We proceed in like manner with this, and each successive quotient, till the whole is reduced to the required denomination.

The last quotient, with the remainders (when there are any) annexed, will be the required answer.

2. Reduce 3265 inches to rods, yards, etc.

12	3265 in.	SUGGESTION. — In dividing by $5\frac{1}{2}$ it is
3	272 ft. 1 in.	usually more convenient to reduce the
$5\frac{1}{2}$	90 yd. 2 ft.	divisor to halves and the dividend also to halves before dividing.
	2	It should be remembered then, that
11	180 [or 2 yd.	the remainder, if there is any, is <i>half-yards</i> , which must be changed to yards
	16 rd. 4 half-yd.,	

Reduce:

3. 4213 gi. to gal., etc.

6. 3539 in. to rd., etc.

4. 3185 pt. to gal., etc.

7. 2456 in. to rd., etc.

5. 5724 gi. to gal., etc.

8. 1605 in. to rd., etc.

- | | |
|----------------------------|-----------------------------------|
| 9. 3564 pt. to bu., etc. | 18. 9894 min. to da., etc. |
| 10. 4683 pt. to bu., etc. | 19. 3943 sq. in. to sq. yd., etc. |
| 11. 3947 pt. to bu., etc. | 20. 5937 sq. in. to sq. yd., etc. |
| 12. 5164 oz. to cwt., etc. | 21. 1894 sheets to R., etc. |
| 13. 8127 oz. to cwt., etc. | 22. 2315 sheets to R., etc. |
| 14. 5983 lb. to tons, etc. | 23. 7864 cu. in. to cu. ft., etc. |
| 15. 3795 lb. to tons, etc. | 24. 6754 gr. to lb. Troy, etc. |
| 16. 4285 hr. to wk., etc. | 25. 8386 gr. to lb. Troy, etc. |
| 17. 8168 sec. to hr., etc. | 26. 9138 gr. to lb. Apoth., etc. |

27. Reduce 2 qt. 1 pt. to the fraction of a gallon.

EXPLANATION. — 2 qt. 1 pt. = 5 pt. ; 1 gal. = 8 pt. Therefore 2 qt. 1 pt., or 5 pt., are $\frac{5}{8}$ of 1 gal., or 8 pt.

- | |
|---|
| 28. Reduce 1 ft. 3 in. to the fraction of a yard. |
| 29. Reduce 3 qt. 1 pt. to the fraction of a peck. |
| 30. Reduce 4 pk. 3 qt. to the fraction of a bushel. |
| 31. Reduce 2 qt. 1 pt. 3 gi. to the fraction of a gallon. |
| 32. Reduce 2 pk. 4 qt. 1 pt. to the fraction of a bushel. |
| 33. Reduce 4 yd. 2 ft. 6 in. to the fraction of a rod. |

ADDITION OF DENOMINATE NUMBERS.

- 264.** 1. Find the sum of 3 gal. 3 qt. 1 pt. ; 5 gal. 2 qt. 1 pt. ; 6 gal. 2 qt. 1 pt. ; 8 gal. 3 qt.

gal.	qt.	pt.
3	3	1
5	2	1
6	2	1
8	3	
<hr/>		
24	3	1

EXPLANATION. — The numbers are written so that units of the same denomination stand in the same column, and we begin at the lowest denomination to add.

The sum of the pints is 3 pt., which are equal to 1 qt. 1 pt. ; the 1 pt. is written under pints and the 1 qt. reserved to add to the quarts.

The sum of the quarts is 11 qt., which are equal to 2 gal. 3 qt. ; the 3 qt. are written under the quarts and the 2 gal. reserved to add to the gallons.

The sum of the gallons is 24 gal., which are written under the gallons.

Find the sum of:

2. 3 gal. 3 qt. 1 pt.; 18 gal. 2 qt. 1 pt.; 12 gal. 3 qt.
3. 8 bu. 3 pk. 5 qt.; 16 bu. 2 pk. 7 qt.; 12 bu. 3 pk. 4 qt.
4. 7 da. 5 hr. 20 min.; 8 da. 14 hr. 10 min.; 6 da. 18 hr.
5. 24 lb. 6 oz. (Av.); 35 lb. 14 oz.; 26 lb. 10 oz.; 83 lb.
6. 13 lb. 4 oz. 10 pwt. 8 gr.; 11 lb. 5 oz. 11 pwt. 16 gr.;
10 lb. 10 oz. 15 pwt. 21 gr.
7. 10 yd. 2 ft. 10 in.; 15 yd. 1 ft. 9 in.; 8 yd. 2 ft. 7 in.;
18 yd. 1 ft. 11 in.; 16 yd. 2 ft. 8 in.
8. 15 gal. 3 qt. 1 pt. 3 gi.; 18 gal. 2 qt. 1 pt. 2 gi.; 28
gal. 2 qt.; 21 gal. 1 pt. 3 gi.
9. 12 A. 35 sq. rd.; 14 A. 110 sq. rd.; 15 A. 132 sq. rd.;
11 A. 96 sq. rd.; 25 A. 100 sq. rd.
10. 5 T. 6 cwt. 14 lb. 10 oz.; 7 T. 15 cwt. 36 lb. 15 oz.;
17 T. 8 cwt. 84 lb. 12 oz.; 10 T. 9 cwt. 94 lb. 11 oz.
11. 5 yd. 2 ft. 8 in.; 4 yd. 1 ft. 11 in.; 4 yd. 2 ft. 9 in.;
3 yd. 1 ft.; 4 yd. 8 in.; $\frac{3}{4}$ of a yard and $\frac{3}{4}$ of a foot.

SUBTRACTION OF DENOMINATE NUMBERS.

265. 1. From 12 lb. 6 oz. 7 pwt. take 3 lb. 8 oz. 10 pwt.

lb.	oz.	pwt.	EXPLANATION.—The numbers are written so that
12	6	7	units of the same denomination stand in the same
3	8	10	column, and we begin at the lowest denomination to
8	9	17	subtract. Since 10 pwt. cannot be subtracted from
			7 pwt., a unit of the next higher order is united with
			the 7 pwt., making 1 oz. 7 pwt., or 27 pwt. Then 27 pwt. — 10 pwt. =
			17 pwt., which we write under pwt. in the remainder.

Since 1 oz. was united with the 7 pwt., only 5 oz. are left in the minuend. Inasmuch as 8 oz. cannot be subtracted from 5 oz., a unit of the next higher order is united with it, making 1 lb. 5 oz., or 17 oz. Then 17 oz. — 8 oz. = 9 oz., which we write under oz. in the remainder.

Since 1 lb. was changed to ounces, there are but 11 lb. left in the minuend. Then 11 lb. — 3 lb. = 8 lb.

Therefore, the remainder is 8 lb. 9 oz. 17 pwt.

2. From 8 bu. 4 pk. 3 qt. take 3 bu. 2 pk. 7 qt.

3. From 12 gal. 2 qt. 1 pt. 2 gi. take 5 gal. 3 qt. 1 pt. 3 gi.
4. From 10 da. 5 hr. 20 min. take 7 da. 7 hr. 10 min.
5. From 31 lb. 4 oz. 15 pwt. take 8 lb. 6 oz. 12 pwt.
6. From 15 yd. 2 ft. 7 in. take 4 yd. 2 ft. 10 in.
7. From 12 rd. 4 yd. 1 ft. take 8 rd. 4 yd. 2 ft.
8. From 13 lb. 8 oz. 5 dr. take 7 lb. 10 oz. 7 dr.
9. From 18 A. 18 sq. rd. take 7 A. 110 sq. rd.
10. From 40 sq. yd. 8 sq. ft. 51 sq. in. take 15 sq. yd. 6 sq. ft. 100 sq. in.
11. From 25 T. 8 cwt. 75 lb. 10 oz. take 10 T. 11 cwt. 35 lb. 15 oz.
12. How many years, months, and days intervened between Dec. 24, 1892, and May 26, 1842?

yr.	mo.	da.	EXPLANATION. — Since the later date expresses
1892	12	24	the greater period of time, it is written as the
1842	5	26	minuend, and the earlier date is, therefore, the
50	6	28	subtrahend. In writing the dates the number of
			the month is given instead of its name. The sub-
			traction is then performed as in denominata numbers, considering a
			month as 30 days, and a year as 12 months.

13. How long was it from Dec. 10, 1835, to May 15, 1887?
14. How long was it from June 13, 1864, to Jan. 1, 1890?
15. How long was it from Apr. 8, 1839, to July 7, 1891?
16. How long was it from Sept. 23, 1881, to Aug. 10, 1892?

MULTIPLICATION OF DENOMINATE NUMBERS.

266. 1. Multiply 3 bu. 3 pk. 7 qt. by 7.

bu.	pk.	qt.	EXPLANATION. — 7 times 7 qt. are 49 qt., or 6 pk.
3	3	7	1 qt. The 1 qt. is written under qt. in the product,
		7	and the 6 pk. reserved to add to the pecks. 7 times
27	3	1	3 pk. are 21 pk.; 21 pk. + 6 pk. reserved are 27 pk.,
			or 6 bu. 3 pk. The 3 pk. are written under pk. in
			the product, and the 6 bu. reserved to add to the bushels.

Seven times 3 bu. are 21 bu.; 21 bu. + 6 bu. reserved are 27 bu.
Therefore, the product is 27 bu. 3 pk. 1 qt.

2. Multiply 4 bu. 3 pk. 5 qt. by 7.
3. Multiply 5 gal. 1 qt. 3 pt. 2 gi. by 6.
4. Multiply 5 lb. 6 oz. 10 pwt. 8 gr. by 8.
5. Multiply 7 lb. 8 oz. 5 dr. 2 sc. 10 gr. by 7.
6. Multiply 3 hr. 20 min. 35 sec. by 5.
7. Multiply 2 T. 5 cwt. 48 lb. 15 oz. by 8.
8. Multiply 2 rd. 3 yd. 2 ft. 10 in. by 9.
9. Multiply 12 cu. yd. 15 cu. ft. 1115 cu. in. by 6.
10. Multiply 8 sq. yd. 2 sq. ft. 45 sq. in. by 5.
11. Multiply 5 da. 8 hr. 15 min. 25 sec. by 4.
12. Multiply 12 rd. 4 yd. 2 ft. 8 in. by 8.
13. Multiply 5 R. 8 qr. 16 sheets by 9.
14. Multiply 6 bar. 10 gal. 3 qt. 1 pt. by 7.

DIVISION OF DENOMINATE NUMBERS.

- 267.** 1. Divide 14 gal. 3 qt. 1 pt. by 4.

gal.	qt.	pt.	EXPLANATION.—
4)	14	3 1	One fourth of 14 gal. is 3 gal. and 2 gal. remainder. The 3 gal. are written in the quotient, and the 2 gal. remainder are united with the 3 qt., making 11 qt.

One fourth of 11 qt. is 2 qt. and 3 qt. remainder. The 2 qt. are written in the quotient, and the 3 qt. remainder united with the 1 pt., making 7 pt.

One fourth of 7 pt. is $1\frac{1}{4}$ pt.

Therefore, the quotient is 3 gal. 2 qt. $1\frac{1}{4}$ pt.

2. Divide 21 gal. 3 qt. 1 pt. 3 gi. by 6.
3. Divide 15 bu. 3 pk. 5 qt. 1 pt. by 4.
4. Divide 13 yd. 2 ft. 6 in. by 7.
5. Divide 23 cwt. 68 lb. 10 oz. by 5.
6. Divide 42 lb. 8 oz. 15 pwt. by 9.
7. Divide 22 lb. 7 oz. 5 dr. 2 sc. by 8.
8. Divide 19 hr. 10 min. 36 sec. by 6.
9. Divide 21 sq. yd. 7 sq. ft. 45 sq. in. by 4.
10. Divide 15 rd. 4 yd. 2 ft. 8 in. by 5.

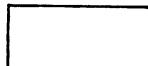
PRACTICAL MEASUREMENTS.

268. To compute the area of rectangular surfaces.

1. What is a square inch? Draw a figure 6 in. long and 1 in. wide with all its angles equal. How many sq. in. does it contain?
2. If the figure were 2 in. wide, how many sq. in. would it contain? How many if it were 3 in. wide? 4 in. wide? 5 in. wide? 6 in. wide?
3. What is a square foot? How many sq. ft. are there in a strip of ground 12 ft. long and 1 ft. wide, with all its angles equal? How many, if it is 2 ft. wide? 3 ft. wide? 4 ft. wide? 5 ft. wide? 6 ft. wide?
4. How many sq. yd. are there in a walk 15 yd. long and 5 yd. wide, having its angles all equal?

269. A figure that has four straight sides and four equal angles is called a **Rectangle**.

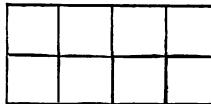
The angles of a rectangle are all right angles.



Rectangle.

270. The number of square units that a surface contains is called its **Area**.

Thus the area of a rectangle 4 in. long and 2 in. wide is 8 sq. in., for it may be divided into 2 rows, each containing 4 sq. in.



For additional instruction in measurements, consult the author's **STANDARD ARITHMETIC**.

WRITTEN EXERCISES.

271. 1. What is the area of a rectangular walk 22 ft. long and 5 ft. 6 in. broad?

$$22 \times 5\frac{1}{2} = 121$$

or,

$$264 \times 66 = 17,424$$

$$17,424 \text{ sq. in.} = 121 \text{ sq. ft.}$$

EXPLANATION. — Since the walk is 22 ft. long and $5\frac{1}{2}$ ft. wide, its area in feet is 121 sq. ft.

The same result can be obtained by expressing the length and breadth in inches, and reducing the result obtained to square feet.

272. Hence it is evident that:

The area of any rectangular surface is obtained by finding the product of the numbers expressing its length and breadth.

The length and breadth must be expressed in units of the *same denomination*.

2. What is the area of the floor of a rectangular room 17 ft. long and 23 ft. wide?
3. How many sq. ft. of surface are there in the side wall of a room 16 ft. long and 12 ft. high? How many sq. yd.?
4. The floor of a rectangular room 18 ft. wide and 24 ft. long contains how many sq. yd.?
5. A farm in the form of a rectangle has a length of 160 rd. and a breadth of 120 rd. How many sq. rd. of land does it contain? How many acres?
6. How many acres are there in a rectangular field 60 rd. wide and 80 rd. long?
7. A farmer paid \$2.50 per acre for plowing a rectangular field that was 25 rd. wide and 45 rd. long. What was the entire expense of plowing the field?
8. A plasterer received 23¢ per sq. yd. for plastering the walls and ceiling of a room 18 ft. long, 15 ft. wide, and 12 ft. high, no deduction being made for the openings. How much did he receive?
9. How much will it cost to carpet the floor of the room with carpeting 1 yd. wide at 85¢ per yd.?
10. The farm of Mr. Hudson is 160 rd. long and 150 rd. wide, and he purchases an adjoining farm 160 rd. long and 90 rd. wide. How many acres has he after the purchase?
11. A room 18 ft. wide and 24 ft. long was carpeted with carpeting 1 yd. wide at \$1.25 per yd. How much did it cost?
12. The length of a rectangular field was 80 rd. and its breadth was 60 rd. If half of it was planted with corn and half the remainder with potatoes, how many acres were planted with each?

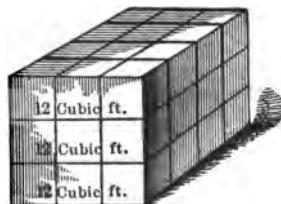
273. To compute the volume of rectangular solids.

1. What is a cubic inch? A cubic foot? A cubic yard?
2. How many cubic feet of stone are there in a rectangular block of stone which is 6 ft. long, 1 ft. wide, and 1 ft. thick?
3. How many cubic feet would there be if the block were 2 ft. wide? 2 ft. thick? 2 ft. wide and 2 ft. thick? How many if it were 7 ft. long, 2 ft. wide, and 2 ft. thick?
4. How many cubic feet are there in a similar block which is 6 ft. long, 3 ft. wide, and 1 ft. thick? In a block 6 ft. long, 3 ft. wide, and 2 ft. thick? In a block 6 ft. long, 3 ft. wide, and 3 ft. thick? In a block 6 ft. long, 4 ft. wide, and 3 ft. thick? In a block 6 ft. long, 3 ft. wide, and 4 ft. thick? In a block 3 ft. long, 4 ft. wide, and 6 ft. thick?

274. The number of solid units that a body contains is its Volume or Solid contents.

The solid units are cubic inches, cubic feet, cubic yards, etc.

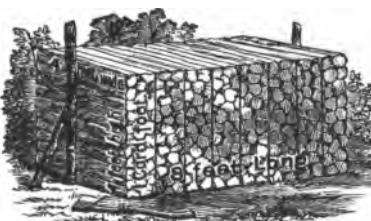
If a solid is 4 ft. long, 3 ft. wide, and 3 ft. thick, its volume is 36 cu. ft., for it may be divided into 3 blocks, each containing 12 cu. ft., as shown by the illustration.



275. A pile of wood or stone 8 ft. long, 4 ft. wide, and 4 ft. high is called a Cord.

A pile that is 1 ft. long, 4 ft. wide, and 4 feet high is called a **Cord Foot**.

Stone walls are sometimes measured by the **Perch**, which contains 24.75 cu. ft.



WRITTEN EXERCISES.

- 276.** 1. How many cubic feet of stone are there in a rectangular block 8 ft. long, 5 ft. 3 in. wide, and 3 ft. 6 in. thick?

$8 \times 5\frac{1}{4} \times 3\frac{1}{2} = 147$ EXPLANATION.—Since a block 8 ft. long, 1 ft. wide, and 1 ft. thick contains 8 cu. ft., a block 5 $\frac{1}{4}$ ft. wide and 1 ft. thick contains 5 $\frac{1}{4}$ times 8 cu. ft., or 42 cu. ft.

Since a block 8 ft. long, 5 $\frac{1}{4}$ ft. wide, and 1 ft. thick, contains 42 cu. ft., a similar block, that is 3 $\frac{1}{2}$ ft. thick, contains 3 $\frac{1}{2}$ times 42 cu. ft., or 147 cu. ft.

- 277.** Hence it is evident that:

The volume or solid contents of a rectangular solid are obtained by finding the product of the numbers expressing its length, breadth, and thickness.

The length, breadth, and thickness must be expressed in units of the same denomination.

Find the volume of solids with dimensions as follows:

2. Length 10 ft., breadth 7 ft., thickness 6 ft.
3. Length 9 in., breadth 11 in., thickness 8 in.
4. Length 5 yd., breadth 8 yd., thickness 10 yd.
5. Length 14 ft., breadth 10 ft., thickness 15 ft.
6. Length 8 ft. 6 in., breadth 8 ft., thickness 10 ft.
7. Length 12 ft. 4 in., breadth 9 ft., thickness 8 ft.
8. Length 10 ft. 6 in., breadth 10 ft., thickness 12 ft.
9. Length 12 ft. 4 in., breadth 10 ft., thickness 8 ft.
10. Length 8 ft. 4 in., breadth 10 ft. 6 in., thickness 8 ft.

How many cords of wood are there in the following piles?

11. Length 24 ft., breadth 4 ft., height 4 ft.
12. Length 32 ft., breadth 4 ft., height 8 ft.
13. Length 48 ft., breadth 16 ft., height 12 ft.
14. How much will the wood in the last pile cost at \$5.50 per cord?

15. How much will it cost to excavate a cellar which is 40 ft. long, 22 ft. wide, and 8 ft. deep, at 30¢ per cubic yard?

16. How much will be the expense of laying a stone wall 40 ft. long, 1 ft. 6 in. thick, and 6 ft. high, at \$2.50 per perch?

278. To measure lumber.

In measuring lumber, boards 1 inch thick or less are estimated by the number of square feet of surface.

Thus a board 1 foot wide and 10 feet long contains 10 feet *board measure*, if it is 1 inch or less in thickness.

The length and width must both be expressed in feet.

279. When lumber is more than 1 inch in thickness, the number of feet board measure is obtained as follows:

The length in feet is multiplied by the breadth in feet, and this product by the number of inches in thickness.

Thus, the number of feet *board measure* in a timber 12 ft. long, 15 in. wide, and 2½ in. thick is obtained as follows:

$$12 \times 1\frac{1}{4} \times 2\frac{1}{2} = 37\frac{1}{2}, \text{ the number of feet board measure.}$$

How many feet are there in the following boards:

1. 16 ft. long, 18 in. wide. 4. 20 ft. long, 14 in. wide.
2. 12 ft. long, 15 in. wide. 5. 15 ft. long, 10 in. wide.
3. 18 ft. long, 9 in. wide. 6. 20 ft. long, 16 in. wide.
7. What will be the cost of 10 boards, each 12 ft. long and 15 in. wide, at \$20 per thousand feet?
8. How many feet board measure are there in 20 joists 18 ft. long, 16 in. wide, and 3 in. thick?
9. What will those joists cost at \$18 per thousand feet?
10. What will be the cost of 20 planks 16 ft. long, 14 in. wide, and 2 in. thick, at \$22 per thousand feet?
11. What will be the cost of 20 planks 12 ft. long, 15 in. wide, and 1½ in. thick, and 40 scantlings 6 in. wide, 12 ft. long and 4 in. thick, at \$18 per thousand feet?

INTEREST.

280. When a person borrows money, he usually has to pay a certain number of *hundredths*, or a certain *per cent*, of the sum for the use of it for each year.

281. The character $\%$ is often used instead of the words *hundredths* or *per cent*.

Thus, 5 hundredths, or 5 per cent, is expressed 5%.

ORAL EXERCISES.

282. 1. When a sum equal to 6% of the money loaned is paid for the use of it for each year, how much is paid for the use of \$100 for 1 yr., or what is the *interest* for 1 yr.?

2. When the rate of interest is 5% per year, how much will be the interest of \$100 for 1 yr.? For 2 yr.? For 3 yr.?

3. When the rate of interest is 8% per year, what will be the interest of \$100 for 1 yr.? \$200? \$500?

4. When the rate of interest is 6% per year, what will be the interest of \$400 for 1 yr.? For 2 yr.? For $2\frac{1}{2}$ yr.? For $\frac{1}{2}$ yr.?

5. When the rate of interest is 4% per year, how much will \$200 amount to in 1 yr.? In 2 yr.? In $2\frac{1}{2}$ yr.?

6. When \$400 is loaned for 3 yr. at 5% per year, how much is the interest? What is the amount due?

What is the interest of:

- | | |
|---------------------------|----------------------------|
| 7. \$600 for 1 yr. at 5%? | 9. \$800 for 2 yr. at 4%? |
| 8. \$500 for 1 yr. at 6%? | 10. \$400 for 3 yr. at 5%? |

11. \$ 600 for $1\frac{1}{2}$ yr. at 3%? 15. \$ 800 for $1\frac{1}{2}$ mo. at 6%?
 12. \$ 200 for $2\frac{1}{2}$ yr. at 5%? 16. \$ 600 for 1 mo. 10 da. at 6%?
 13. \$ 800 for $\frac{1}{2}$ yr. at 6%? 17. \$ 600 for 2 mo. 10 da. at 6%?
 14. \$ 600 for 1 mo. at 6%? 18. \$ 800 for 3 yr. at 5%?

283. The sum paid for the use of money is called **Interest**.

284. The sum for the use of which interest is paid is called the **Principal**.

285. The sum of the principal and interest is called the **Amount**.

286. In computing interest it is usual to regard a year as 12 months, and a month as 30 days.

WRITTEN EXERCISES.

287. 1. What is the interest of \$ 420.50 for 5 yr. at 5%?

SOLUTION.

\$ 420.50 Principal.

.05 Rate.

\$ 21.0250 Interest for 1 yr.

5

\$ 105.1250 Interest for 5 yr.

Find the interest of:

2. \$ 212.34 for 3 yr. at 4%. 5. \$ 219.20 for 4 yr. at 7%.

3. \$ 426.12 for 4 yr. at 5%. 6. \$ 324.16 for 5 yr. at 4%.

4. \$ 324.25 for 5 yr. at 6%. 7. \$ 582.30 for 6 yr. at 5%.

8. What is the interest of \$ 366.20 for 3 yr. 8 mo. at 6%?

SOLUTION.

\$ 366.20

.06

\$ 21.9720, int. for 1 yr.

3

\$ 65.9160, int. for 3 yr.

$\frac{1}{2}$ of the int. for 1 yr. = 10.9860, int. for 6 mo.

$\frac{1}{2}$ of the int. for 6 mo. = 3.6620, int. for 2 mo.

\$ 80.5640, int. for 3 yr. 8 mo.

Find the interest of:

9. \$412.20 for 2 yr. 8 mo. at 4%.
10. \$318.36 for 3 yr. 6 mo. at 6%.
11. \$425.24 for 4 yr. 5 mo. at 5%.
12. \$685.60 for 3 yr. 7 mo. at 7%.
13. \$437.28 for 5 yr. 2 mo. at 8%.
14. \$314.26 for 4 yr. 4 mo. at 3%.
15. \$736.16 for 2 yr. 10 mo. at 6%.

16. Find the amount of \$524.36 for 2 yr. 5 mo. 18 da. at 7%.

SOLUTION.

$$\begin{array}{r}
 \$524.36 \\
 \cdot07 \\
 \hline
 \$36.7052, \text{ int. for 1 yr.} \\
 \hline
 2
 \end{array}$$

\$73.4104, int. for 2 yr.

$$\begin{array}{r}
 \frac{1}{2} \text{ of the int. for 1 yr.} = 12.2350, \text{ int. for } 4 \text{ mo.} \\
 \frac{1}{2} \text{ of the int. for 4 mo.} = 3.0587, \text{ int. for } 1 \text{ mo.} \\
 \frac{1}{2} \text{ of the int. for 1 mo.} = 1.5293, \text{ int. for } 15 \text{ da.} \\
 \frac{1}{2} \text{ of the int. for 15 da.} = .3058, \text{ int. for } 3 \text{ da.} \\
 \hline
 \end{array}$$

$$\begin{array}{r}
 \$90.5392, \text{ int. for 2 yr. 5 mo. 18 da.} \\
 524.36, \text{ principal.} \\
 \hline
 \$614.8992, \text{ amount.}
 \end{array}$$

Find the interest and amount of:

17. \$360.24 for 2 yr. 7 mo. 15 da. at 5%.
18. \$415.48 for 3 yr. 5 mo. 10 da. at 6%.
19. \$217.36 for 2 yr. 6 mo. 15 da. at 4%.
20. \$423.35 for 3 yr. 8 mo. 15 da. at 6%.
21. \$720.16 for 3 yr. 7 mo. 17 da. at 5%.
22. \$468.36 for 3 yr. 9 mo. 10 da. at 4%.
23. \$219.47 for 2 yr. 5 mo. 12 da. at 6%.
24. \$325.34 for 1 yr. 1 mo. 13 da. at 7%.
25. \$436.33 for 4 yr. 2 mo. 10 da. at 5%.
26. \$297.45 for 5 yr. 3 mo. 15 da. at 6%.

GENERAL REVIEW.

- 288.** 1. What will 50 sheep cost if 75 are worth \$375?
2. How many years is it since the Declaration of Independence was signed?
3. A rope 13 yd. 2 ft. 6 in. long was cut into 3 equal parts. How long were they?
4. A farmer sold at one time 3 bu. 3 pk. 7 qt. of clover seed, and at another 5 bu. 2 pk. 6 qt. How much did he sell?
5. A wagon wheel is 15 ft. 8 in. in circumference. How many revolutions will it make in going 5 miles?
6. James spent 57 cents, which was $\frac{3}{8}$ of all the money he had. How much money had he?
7. A father gave his children \$8 apiece. If he had given them \$12 apiece it would have taken \$40 more. How many children were there?
8. The larger of two fractions is $\frac{17}{24}$, and the difference between them is $\frac{2}{3}$. What is the smaller fraction?
9. If $\frac{3}{4}$ of an acre of land produces 125 bushels of potatoes, how many bushels will $3\frac{1}{2}$ acres produce?
10. A can do a piece of work in 10 days, and B can do it in 8 days. In what time can both do it?
11. A man who owned $\frac{2}{3}$ of a factory sold $\frac{1}{2}$ of his share for \$3500. What was the factory worth at that rate?
12. Two boys did some work for 65 cents. If one of them earned 15 cents more than the other, how much did each earn?

13. A man who spent $\frac{1}{4}$ of his money and \$12 more found that he had \$15 left. How much had he at first?
14. A man died leaving to his 3 children a farm of 317 A. 115 sq. rd. to be divided equally among them. What was each one's share of the land?
15. How many acres of land, at \$45 per acre, can I get for 90 oxen at \$37.75 per head?
16. Mr. David Rounds purchased from D. Henderson & Co. 13 yd. muslin @ 8¢, 8 yd. sheeting @ \$.11 $\frac{1}{2}$, 25 yd. ribbon @ 7¢ per yd., 10 lb. cotton batting at 8 $\frac{1}{2}$ ¢, 2 pr. gloves @ \$1.25, 3 doz. pearl buttons @ \$.27. Make out a receipted bill.
17. How long will it take a person to count \$1,000,000 if he counts \$5 per second for 10 hours per day?
18. If a railroad train runs at the rate of 52 $\frac{1}{2}$ mi. per hour, how far will it run in 8 hr. 40 min.?
19. How many feet board measure are there in 45 joists, each 18 ft. long, 16 in. wide, and 2 in. thick?
20. A farmer sold 3 T. 18 cwt. 85 lb. of hay at \$9.50 per ton. How much did he get for it?
21. A cask of wine contained 58 gal. 3 qt. 1 pt. How much was it worth at \$2.75 per gallon?
22. A silversmith sold some silver plate at \$1.50 per ounce, receiving for it \$93. How much did he sell?
23. A merchant had 320 yd. of cloth, which he made into suits which required 6 yd. 2 ft. each. How many suits did he make?
24. What will be the expense of painting a gable roof 48 ft. long and 25 ft. wide at \$.30 per sq. yd.?
25. How many yards of matting, 1 $\frac{1}{2}$ yd. wide, will be required to cover a floor 7 yd. 2 ft. 7 $\frac{1}{2}$ in. wide, and 18 ft. long? What will be the cost of it at \$.70 per lineal yd.?
26. A man has a lot containing 10 acres that is 25 rods wide. How long is it?

27. A money-lender loaned \$ 520 for two years at 6%. How much money should he receive as principal and interest at the end of the time?
28. One number is $\frac{2}{3}$ of another, and their sum is 490. What are the numbers?
29. A man who owned $\frac{1}{3}$ of a vessel sold $\frac{1}{2}$ of his share for \$ 12,500. At that rate, what was the vessel worth?
30. Two men engaged in business, one furnishing $\frac{1}{3}$ of their capital, and the other the rest. If they gained \$ 1477, what was each one's share of the gain?
31. Mr. Davis loaned his nephew \$ 350 on Jan. 1, 1890, at 6% interest. If the debt was paid Jan. 1, 1893, how much was the amount?
32. How much will it cost to carpet a room 18 ft. wide and 24 ft. long at \$ 1.15 per sq. yd.?
33. A druggist sold 1 lb. of quinine, which was made into two-grain pills. How many pills were there?
34. A stationer bought 10 reams of paper at \$ 1.90 per ream, and sold it at 1 cent per sheet. How much was his gain?
35. Henry bought at a grocer's 15 lb. 12 oz. of butter at 32 cents per pound. How much did it cost him?
36. Two brothers earned during their vacation \$ 110, but one of them earned only $\frac{2}{3}$ as much as the other. How much did each earn?
37. If $\frac{1}{4}$ of a bbl. of flour costs \$ 4.20, what will 7 bbl. cost?
38. Divide $\frac{2}{3}$ of $\frac{3}{4}$ of $\frac{4}{5}$ of $\frac{10}{11}$ by $\frac{1}{2}$ of $\frac{12}{13}$ of $\frac{7}{8}$ of $\frac{9}{10}$.
39. Sound travels about 1140 feet per second. If a flash of lightning is seen, but the thunder accompanying it is not heard until 35 seconds later, how far away is the lightning?
40. The product of two numbers is 3.35, and one of them is .25. What is the other number?

ANSWERS.

- Page 90.** — **2.** 283. **3.** 254. **4.** 231. **5.** 212. **6.** 138. **7.** 185.
8. 300. **9.** 198. **10.** 198. **11.** 280. **12.** 315. **13.** 274. **14.** 213.
15. 152. **16.** 107. **17.** 280. **18.** 226. **19.** 289. **20.** 233. **21.** 313.
22. 298. **23.** 197. **24.** 179. **25.** 271.
- Page 91.** — **26.** 46 mi. **27.** \$57. **28.** 88 yr. **29.** 100 articles.
30. 38 boys; 41 girls; 79 children. **31.** 134 fishes. **32.** 180 pounds.
33. 262 pages. **34.** 168 acres. **35.** 31 mi. **36.** 44 examples. **37.** 79¢.
38. 87 bu.
- Page 92.** — **39.** 50 chickens. **40.** \$57. **41.** 98%. **42.** \$1.32.
43. 105 cows. **44.** \$1.83. **45.** 112 things. **46.** 126 marbles.
47. 97 leaves. **48.** 121 loaves. **49.** 262 qt. **50.** 148 lb.
- Page 93.** — **51.** 88 animals. **52.** \$25. **53.** \$40. **54.** 252. **55.** 266
yd. **56.** \$88, horse; \$176, together. **57.** \$1.30. **58.** 123 lines.
59. \$1821. **60.** 1335 acres. **61.** 3198 pupils in all; 1401, 4th school.
- Page 94.** — **62.** 593. **63.** 470. **64.** 541. **65.** 467. **66.** 461. **67.** 365.
68. 542. **69.** 5116. **70.** 5048. **71.** 3681. **72.** 4129. **73.** 5415.
74. 5311. **75.** 71,425. **76.** 67,053. **77.** 64,794. **78.** 48,551.
79. 23,437.
- Page 96.** — **2.** 532. **3.** 723. **4.** 351. **5.** 413. **6.** 214. **7.** 441.
8. 633. **9.** 201. **10.** 515. **11.** 543. **12.** 410. **13.** 662. **14.** 126.
15. 456. **16.** 634. **17.** 613. **18.** 245. **19.** 724. **20.** 654.
- Page 97.** — **24.** 17. **25.** 17. **26.** 37. **27.** 23. **28.** 41. **29.** 38.
30. 16. **31.** 23. **32.** 54. **33.** 24. **34.** 34. **35.** 48. **36.** 3. **37.** 68.
38. 55. **39.** 37. **40.** 614. **41.** 414. **42.** 222. **43.** 27. **44.** 513.
45. 223. **46.** 110. **47.** 208. **48.** 129. **49.** 126. **50.** 317. **51.** 428.
52. 781. **53.** 292. **54.** 221.
- Page 98.** — **55.** 166. **56.** 323. **57.** 468. **58.** 227. **59.** 414. **60.** 601.
61. 209. **62.** 153. **63.** 625. **64.** 434. **65.** 657. **66.** 609. **67.** 641.
68. \$49. **69.** \$721. **70.** 109 children. **71.** 281 eggs. **72.** 758 birds.
73. 79 pints. **74.** 115 horses. **75.** 265 cows. **76.** \$432. **77.** 581 bu.
78. 635 apples. **79.** 365 da. **80.** 56 yr. **81.** \$148. **82.** \$188.
- Page 99.** — **1.** 534 gal. **2.** 545 mi. **3.** 3621 bricks. **4.** Lost \$22.
5. 594. **6.** \$195. **7.** 221 mi. **8.** Answers differ. **9.** 836 yd.
10. \$5669, entire cost; \$2375 more. **11.** Answers differ. **12.** 112 ft.
- Page 100.** — **13.** \$289. **14.** 451 apple trees; 715 trees. **15.** 34 bu.
16. \$97.49. **17.** \$2.87. **18.** \$89. **19.** \$2058; \$431.50. **20.** \$1262.
21. 237 mi. **22.** 618 mi. **23.** 108 pupils. **24.** \$556. **25.** 58%.
- Page 101.** — **26.** 375. **27.** 225 pages. **28.** \$1.00. **29.** 430 pupils.
30. 207 bu. **31.** \$3.25. **32.** \$101.11. **33.** \$351.60. **34.** \$104.64.

35. \$90.64. 36. \$11.65. 37. \$34.99. 38. \$86.23. 39. \$44.73.

40. \$109.95. 41. \$77.91.

Page 107.—2. 230. 3. 222. 4. 196. 5. 558. 6. 602. 7. 392.
 8. 207. 9. 464. 10. 387. 11. 510. 12. 957. 13. 1104. 14. 2905.
 15. 2976. 16. 4230. 17. 1456. 18. 2754. 19. 1625. 20. 7776.
 21. 1300. 22. 4806. 23. 6524. 24. 5709. 25. 10,044. 26. 12,980.
 27. 14,305. 28. 11,826. 29. 34,122. 30. 30,672. 31. 21,116.
 32. 73,512. 33. 27,538. 34. 27,340. 35. 6,346. 36. 47,124.
 37. 23,844. 38. 26,400 ft. 39. 195 yd. 40. \$187.50. 41. \$66.00.
 42. 376 mi. 43. \$426.25. 44. \$190.80.

Page 108.—45. \$630.00. 46. \$91.75. 47. \$58,600. 48. \$25,722.00.
 49. \$71,667. 50. 50,592 papers. 51. \$698.80. 52. \$832.95. 53. 1000
 tons. 54. 230,370 passengers. 55. \$269,465. 56. \$1921.35. 57. 83,208.
 58. 22,648 mi.

Page 109.—59. 68,355 bunches. 60. 11,738; 17,807; 23,476;
 29,345; 35,214; 41,083; 46,952; 52,821. 61. 14,592; 21,888; 29,184;
 36,480; 43,776; 51,072; 58,368; 65,664. 63. 16,328; 24,492; 32,656;
 40,820; 48,984; 57,148; 65,312; 73,476. 63. 7908; 11,862; 15,816;
 19,770; 23,724; 27,678; 31,632; 35,586. 64. 7714; 11,571; 15,428;
 19,285; 23,142; 26,999; 30,856; 34,713. 65. 11,358; 17,037; 22,716;
 28,395; 34,074; 39,753; 45,432; 51,111. 66. 173,948; 260,919;
 347,892; 434,865; 521,838; 608,811; 695,784; 792,757. 67. 47,094;
 70,641; 94,188; 117,735; 141,282; 164,829; 188,376; 211,923.
 68. 130,264; 195,396; 260,528; 325,660; 390,792; 455,924; 521,056;
 586,188. 69. 37,144; 55,716; 74,288; 92,860; 111,432; 130,004;
 148,576; 167,148. 70. 84,728; 127,092; 169,466; 211,820; 254,184;
 296,548; 338,912; 381,276. 71. 76,906; 115,359; 153,812; 192,265;
 230,718; 269,171; 307,624; 346,077. 72. 98,724; 145,086; 193,448;
 241,810; 290,172; 338,534; 386,896; 435,268. 73. 65,146; 97,719;
 130,292; 162,865; 195,438; 228,011; 260,584; 293,157. 74. 143,370;
 215,055; 286,740; 358,425; 430,110; 501,795; 573,480; 645,165.

Page 110.—1. 450. 2. 670. 3. 380. 4. 460. 5. 760. 6. 32,400.
 7. 41,600. 8. 72,300. 9. 34,200. 10. 51,800. 11. 5,463,000.
 12. 3,784,000. 13. 51,690,000. 14. 42,370,000. 15. 819,300,000.

Page 111.—17. 900. 18. 1110. 19. 2720. 20. 5840. 21. 3360.
 22. 3800. 23. 1500. 24. 2070. 25. 5880. 26. 3900. 27. 173,000.
 28. 83,700. 29. 362,600. 30. 237,600. 31. 154,000. 32. 298,800.
 33. 272,800. 34. 191,400. 35. 345,500. 36. 177,600. 37. 23,676,000.
 38. 21,116,000. 39. 58,555,000. 40. 65,034,000. 41. 43,096,000.
 42. 36,970,000. 43. 8,607,000. 44. 19,998,000. 45. 35,060,000.
 46. 55,092,000.

Page 112.—51. 6408. 52. 22,572. 53. 55,998. 54. 11,742.
 55. 12,555. 56. 40,260. 57. 51,471. 58. 33,432. 59. 12,354.
 60. 69,768. 61. 70,308. 62. 22,240. 63. 59,616. 64. 45,936.
 65. 16,016. 66. 907,420. 67. 2,256,404. 68. 2,237,340. 69. 3,255,112.
 70. 1,640,536. 71. 3,642,690. 72. 1,062,152. 73. 1,330,329.
 74. 6,780,025. 75. 6,938,840. 76. 2,883,686. 77. 8,189,100.
 78. 5,836,005. 79. 4,965,954. 80. 3,468,265. 81. 4,512,816.
 82. 2,749,835. 83. 5,996,046. 84. 4,323,284. 85. 2,673,064.
 86. 4,846,122. 87. 2,353,176. 88. 27,381,900. 89. 29,277,136.

90. 60,355,476. 91. 302,422,875. 92. 468,083,874. 93. 186,173,400.
 94. 307,907,152. 95. 447,832,464.

Page 113. — 96. \$28,189.68. 97. \$14,795.60. 98. \$52,071.30.
 99. \$71,125.80. 100. \$29,647.02. 101. \$2,164,284.20. 102. \$2,053.
 983.00. 103. \$2,884,098.15. 104. \$1,450,467.60. 105. \$2,774,732.64.
 106. \$2808.65. 107. 195,360 ft. 108. \$92,022. 109. 441,066 bricks.
 110. \$16,686.42. 111. \$516.46. 112. 14,208 hides. 113. \$4847.04.
 114. 49,680 pencils. 115. 525,600 min. 116. \$3538.75. 117. 207,360
 times. 118. 3,747,090.

Page 114. — 119. 11,341,784 gal. 120. \$6,356,220. 121. \$444,475.
 122. 160,965 bbl. 123. \$75,209.40.

Page 116. — 1. \$220. 2. \$16,675. 3. \$950. 4. \$8024. 5. \$321.50.
 6. \$184.81. 7. A. \$4918; B. \$4295.

Page 117. — 8. \$243.25. 9. \$653.75. 10. \$332.75. 11. \$10,136.50.
 12. \$1924.25. 13. \$11,340. 14. \$8280. 15. \$77 loss. 16. \$32,810.
 17. \$773.70. 18. \$6840.

Page 118. — 19. \$2526.75. 20. \$31,298.75. 21. \$12,799.60.
 22. 1740 people. 23. \$13,833. 24. 10,720,000 seeds. 25. 252,018,000
 fishes. 26. 147 mi. 27. 123 mi.

Page 123. — 6. 1218. 7. 1366. 8. 594. 9. 496. 10. 1545. 11. 984.
 12. 1078. 13. 876. 14. 454. 15. 166. 16. 125. 17. 211. 18. 554.
 19. 1353. 20. 548.

Page 124. — 21. 1645 $\frac{1}{2}$. 22. 918 $\frac{1}{2}$. 23. \$14.88 $\frac{1}{2}$. 24. \$9.08 $\frac{1}{2}$.
 25. \$10,428. 26. \$14,653. 27. \$8374 $\frac{1}{2}$. 28. \$2296 $\frac{1}{2}$. 29. \$12,052.
 30. \$4857 $\frac{1}{2}$. 31. \$9479 $\frac{1}{2}$. 32. \$10,149. 33. \$12,024. 34. \$2862 $\frac{1}{2}$.
 35. 454. 36. 237. 37. 391. 38. 459. 39. 13,170 $\frac{1}{2}$. 40. 17,897.
 41. 4390 $\frac{1}{2}$. 42. 450 coats. 43. 211 weeks. 44. \$753. 45. 156 hr.
 46. 392 hr. 47. 689 people. 48. 3271 yd. 49. 6523 cloaks. 50. 586
 fathoms. 51. 650; 487 $\frac{1}{2}$; 390; 325; 278 $\frac{1}{2}$; 243 $\frac{1}{2}$; 216 $\frac{1}{2}$. 52. 492; 369;
 295 $\frac{1}{2}$; 248; 210 $\frac{1}{2}$; 184 $\frac{1}{2}$; 164. 53. 1453 $\frac{1}{2}$; 1090; 872; 726 $\frac{1}{2}$; 622 $\frac{1}{2}$;
 545; 484 $\frac{1}{2}$. 54. 981; 735 $\frac{1}{2}$; 588 $\frac{1}{2}$; 490 $\frac{1}{2}$; 420 $\frac{1}{2}$; 387 $\frac{1}{2}$; 327. 55. 1872;
 1404; 1123 $\frac{1}{2}$; 936; 802 $\frac{1}{2}$; 702; 624. 56. 2303; 1727 $\frac{1}{2}$; 1381 $\frac{1}{2}$; 1151 $\frac{1}{2}$;
 987; 863 $\frac{1}{2}$; 767 $\frac{1}{2}$. 57. 872; 654; 523 $\frac{1}{2}$; 436; 373 $\frac{1}{2}$; 327; 290 $\frac{1}{2}$.
 58. 628; 471; 376 $\frac{1}{2}$; 314; 269 $\frac{1}{2}$; 235 $\frac{1}{2}$; 209 $\frac{1}{2}$. 59. 5762; 4321 $\frac{1}{2}$;
 3457 $\frac{1}{2}$; 2881; 2469 $\frac{1}{2}$; 2160 $\frac{1}{2}$; 1920 $\frac{1}{2}$. 60. 3871 $\frac{1}{2}$; 2903 $\frac{1}{2}$; 2323; 1935 $\frac{1}{2}$;
 1650 $\frac{1}{2}$; 1451 $\frac{1}{2}$; 1290 $\frac{1}{2}$. 61. 14,951 $\frac{1}{2}$; 11,213 $\frac{1}{2}$; 8970 $\frac{1}{2}$; 7475 $\frac{1}{2}$; 6407 $\frac{1}{2}$.
 5606 $\frac{1}{2}$; 4983 $\frac{1}{2}$. 62. 6269 $\frac{1}{2}$; 4702 $\frac{1}{2}$; 3761 $\frac{1}{2}$; 3134 $\frac{1}{2}$; 2687; 2351 $\frac{1}{2}$; 2089 $\frac{1}{2}$.
 63. 92,407 $\frac{1}{2}$; 69,305 $\frac{1}{2}$; 55,444 $\frac{1}{2}$; 46,203 $\frac{1}{2}$; 39,603 $\frac{1}{2}$; 34,652 $\frac{1}{2}$; 30,802 $\frac{1}{2}$.
 64. 182,779; 137,084 $\frac{1}{2}$; 109,667 $\frac{1}{2}$; 91,389 $\frac{1}{2}$; 78,333 $\frac{1}{2}$; 68,542 $\frac{1}{2}$; 60,926 $\frac{1}{2}$.
 65. 129,882 $\frac{1}{2}$; 97,412; 77,929 $\frac{1}{2}$; 64,941 $\frac{1}{2}$; 55,664; 48,706; 43,294 $\frac{1}{2}$.
 66. 87,316 $\frac{1}{2}$; 65,487 $\frac{1}{2}$; 52,390; 43,658 $\frac{1}{2}$; 37,421 $\frac{1}{2}$; 32,743 $\frac{1}{2}$; 29,105 $\frac{1}{2}$.

Page 126. — 4. 372. 5. 486. 6. 387 $\frac{1}{2}$. 7. 426 $\frac{1}{2}$. 8. 384 $\frac{1}{2}$.
 9. 573 $\frac{1}{2}$. 10. 425 $\frac{1}{2}$. 11. 298 $\frac{1}{2}$. 12. 342 $\frac{1}{2}$. 13. 513 $\frac{1}{2}$.
 14. 52 $\frac{1}{2}$. 15. 32 $\frac{1}{2}$. 16. 97 $\frac{1}{2}$. 17. 54 $\frac{1}{2}$. 18. 69 $\frac{1}{2}$.
 20. 341 $\frac{1}{2}$. 21. 243 $\frac{1}{2}$. 22. 81 $\frac{1}{2}$. 23. 49 $\frac{1}{2}$. 24. 329 $\frac{1}{2}$. 25. 95 $\frac{1}{2}$.
 26. 58 $\frac{1}{2}$. 27. 68 $\frac{1}{2}$. 28. 166 $\frac{1}{2}$. 29. 39 $\frac{1}{2}$. 30. 96 $\frac{1}{2}$. 31. 135 $\frac{1}{2}$.

Page 128. — 36. 382. 37. 213. 38. 66. 39. 48. 40. 55. 41. 73.

42. 456. **43.** 144. **44.** 67. **45.** 82. **46.** 87. **47.** 36. **48.** 1829 $\frac{1}{4}$; 1200 $\frac{1}{2}$; 893 $\frac{1}{4}$; 711 $\frac{1}{4}$; 591 $\frac{1}{2}$. **49.** 3118 $\frac{1}{4}$; 2046 $\frac{1}{4}$; 1522 $\frac{1}{4}$; 1212 $\frac{1}{4}$; 1007 $\frac{1}{2}$. **50.** 4507 $\frac{1}{4}$; 2957 $\frac{1}{2}$; 2201 $\frac{1}{4}$; 1752 $\frac{1}{4}$; 1456 $\frac{1}{2}$. **51.** 2175 $\frac{1}{2}$; 1427 $\frac{1}{2}$; 1062 $\frac{1}{4}$; 845 $\frac{1}{4}$; 702 $\frac{1}{4}$. **52.** 4034 $\frac{1}{4}$; 2847 $\frac{1}{4}$; 1970 $\frac{1}{4}$; 1569 $\frac{1}{4}$; 1303 $\frac{1}{2}$. **53.** 3064 $\frac{1}{4}$; 2004 $\frac{1}{2}$; 1491 $\frac{1}{4}$; 1187 $\frac{1}{4}$; 986 $\frac{1}{4}$. **54.** 4755 $\frac{1}{4}$; 3121 $\frac{1}{4}$; 2322 $\frac{1}{4}$; 1849 $\frac{1}{4}$; 1536 $\frac{1}{4}$. **55.** 979 $\frac{1}{4}$; 642 $\frac{1}{4}$; 478 $\frac{1}{4}$; 380 $\frac{1}{4}$; 316 $\frac{1}{2}$. **56.** 3386 $\frac{1}{4}$; 2222 $\frac{1}{4}$; 1653 $\frac{1}{4}$; 1316 $\frac{1}{4}$; 1094 $\frac{1}{4}$. **57.** 1928 $\frac{1}{4}$; 1265 $\frac{1}{2}$; 941 $\frac{1}{4}$; 750 $\frac{1}{4}$; 623 $\frac{1}{4}$. **58.** 1704 $\frac{1}{4}$; 1118 $\frac{1}{4}$; 832 $\frac{1}{4}$; 682 $\frac{1}{4}$; 550 $\frac{1}{4}$. **59.** 646 $\frac{1}{4}$; 424 $\frac{1}{4}$; 315 $\frac{1}{4}$; 251 $\frac{1}{4}$; 208 $\frac{1}{4}$. **60.** 4237 $\frac{1}{4}$; 2781; 2069 $\frac{1}{4}$; 1648; 1369 $\frac{1}{4}$. **61.** 2229 $\frac{1}{4}$; 1463 $\frac{1}{4}$; 1088 $\frac{1}{4}$; 867 $\frac{1}{4}$; 720 $\frac{1}{4}$. **62.** 4566 $\frac{1}{2}$; 2996 $\frac{1}{4}$; 2230 $\frac{1}{4}$; 1775 $\frac{1}{4}$; 1475 $\frac{1}{4}$. **63.** 33,515 $\frac{1}{2}$; 21,994 $\frac{1}{4}$; 16,367 $\frac{1}{4}$; 13,033 $\frac{1}{4}$; 10,828 $\frac{1}{4}$. **64.** 23,453 $\frac{1}{4}$; 15,391 $\frac{1}{4}$; 11,454 $\frac{1}{4}$; 9120 $\frac{1}{2}$; 7577 $\frac{1}{4}$. **65.** 31,859 $\frac{1}{2}$; 20,907 $\frac{1}{4}$; 15,559 $\frac{1}{4}$; 12,389 $\frac{1}{4}$; 10,293 $\frac{1}{4}$. **66.** 35,897 $\frac{1}{2}$; 23,426 $\frac{1}{4}$; 17,433 $\frac{1}{4}$; 13,882 $\frac{1}{4}$; 11,532 $\frac{1}{4}$. **67.** 39,920 $\frac{1}{2}$; 26,197 $\frac{1}{4}$; 19,495 $\frac{1}{4}$; 15,524 $\frac{1}{4}$; 12,897 $\frac{1}{4}$. **68.** 1024 $\frac{1}{4}$; 1353 $\frac{1}{2}$; 1985; 824 $\frac{1}{4}$; 1083. **69.** 970 $\frac{1}{4}$; 1282 $\frac{1}{4}$; 1890 $\frac{1}{4}$; 780 $\frac{1}{4}$; 1026 $\frac{1}{4}$. **70.** 2665 $\frac{1}{4}$; 3522 $\frac{1}{4}$; 5191 $\frac{1}{4}$; 2144 $\frac{1}{4}$; 2818 $\frac{1}{4}$. **71.** 1302 $\frac{1}{4}$; 1721 $\frac{1}{2}$; 2537 $\frac{1}{4}$; 1048; 1377 $\frac{1}{4}$. **72.** 806 $\frac{1}{4}$; 1065 $\frac{1}{4}$; 1569 $\frac{1}{4}$; 848 $\frac{1}{4}$; 852 $\frac{1}{4}$. **73.** 1374 $\frac{1}{2}$; 1815 $\frac{1}{4}$; 2676 $\frac{1}{4}$; 1105 $\frac{1}{4}$; 1452 $\frac{1}{4}$. **74.** 1960 $\frac{1}{4}$; 2591 $\frac{1}{4}$; 3818 $\frac{1}{4}$; 1577 $\frac{1}{4}$; 2073 $\frac{1}{4}$. **75.** 2596 $\frac{1}{4}$; 3430 $\frac{1}{4}$; 5055 $\frac{1}{4}$; 2088 $\frac{1}{4}$; 2744 $\frac{1}{4}$. **76.** 518 $\frac{1}{4}$; 685 $\frac{1}{4}$; 1010 $\frac{1}{4}$; 417 $\frac{1}{4}$; 548 $\frac{1}{4}$. **77.** 930 $\frac{1}{4}$; 1230; 1812 $\frac{1}{4}$; 748 $\frac{1}{4}$; 984. **78.** 1810 $\frac{1}{4}$; 2392 $\frac{1}{4}$; 3525 $\frac{1}{4}$; 1456 $\frac{1}{4}$; 1913 $\frac{1}{4}$. **79.** 23174 $\frac{1}{4}$; 3062 $\frac{1}{4}$; 4512 $\frac{1}{4}$; 1864 $\frac{1}{4}$; 2449 $\frac{1}{4}$. **80.** 12,698 $\frac{1}{4}$; 16,779 $\frac{1}{4}$; 24,728; 10,213 $\frac{1}{4}$; 13,423 $\frac{1}{4}$. **81.** 7289 $\frac{1}{4}$; 9633; 14,196; 5863 $\frac{1}{4}$; 7706 $\frac{1}{4}$. **82.** 3336 $\frac{1}{4}$; 4409 $\frac{1}{4}$; 6497 $\frac{1}{4}$; 2683 $\frac{1}{4}$; 3527 $\frac{1}{4}$. **83.** 21,019 $\frac{1}{4}$; 27,776 $\frac{1}{4}$; 40,933 $\frac{1}{4}$; 16,907 $\frac{1}{4}$; 22,221 $\frac{1}{4}$. **84.** 541. **85.** 723. **86.** 2767 $\frac{1}{4}$. **87.** 725 $\frac{1}{4}$.

Page 131. — 1. \$61. 2. 23 yd. 3. 248 weeks.

Page 132. — 4. \$130; \$650. 5. 144 doz. 6. \$53.35. 7. 128 trees. 8. 808 times. 9. 301 acres. 10. 2240 lb. 11. 150 da.; 2 $\frac{1}{4}$ da. 12. 72 hr.; 3 da. 13. 1569. 14. 2311, multiplicand. 15. 467. 16. 92 $\frac{1}{4}$ lb.; sister, 83 $\frac{1}{4}$ lb. 17. \$51,614. 18. \$3080. 19. 97 schooners. 20. 23 $\frac{1}{4}$ gal. 21. 259 families.

Page 133. — 22. 1000. 23. 43 cars. 24. 234 da. 25. 5130 shares. 26. 5 $\frac{1}{4}$ mi. 27. 6272. 28. \$224. 29. 352 times. 30. 27 posts; 104 rails. 31. 488 acres. 32. A, \$208; B, \$224. 33. 3535; 3207. 34. 779 gal.

Page 134. — 35. \$4.64. 36. Elsie, \$2.65; Lottie, \$3.05. 37. 14 da. 38. \$584. 39. 629 mi. 40. \$158. 41. \$1816. 42. \$1149.12. 43. 472 bu. 44. 12 horses; 41 sheep. 45. 25 times. 46. 16 da. 47. Gained \$161. 48. 50 gross.

Page 135. — 49. \$75.60. 50. 5 yr. 51. \$98.79. 52. 10 $\frac{1}{4}$ yr. 53. Gained \$40. 54. 938 mi. 55. 98 mi. 56. A, \$46; B, \$53; C, \$99. 57. Howard, 35 fishes; Jamie, 23 fishes; Roy, 19 fishes. 58. \$198. 59. \$1410. 60. \$396.

Page 136.—61. \$4725. 62. B, 442 acres; C, 1284 acres; D, 1511 acres. 63. 76 lb. 64. 8 yr. 65. \$55. 66. 406. 67. 22,962. 68. 42,909.

Page 137.—1. 55. 2. 39. 3. 10. 4. 2. 5. 8. 6. 5. 7. 21. 8. 5. 9. 36. 10. 4. 11. 16. 12. 36. 13. 58. 14. 10. 15. 117. 16. 40. 17. 112. 18. 56. 19. 109. 20. 60. 21. 157. 22. 7.

Page 141.—2. 2, 2, 2, 3, 7. 3. 2, 2, 2, 2, 2, 7. 4. 2, 2, 2, 2, 3, 3. 5. 2, 3, 3, 11. 6. 2, 2, 2, 3, 3, 3. 7. 2, 2, 11, 11. 8. 2, 2, 2, 2, 2, 2, 3, 3. 9. 2, 2, 2, 2, 3, 3, 3. 10. 2, 2, 2, 2, 3, 7. 11. 2, 2, 3, 3, 11. 12. 5, 5, 5, 5. 13. 2, 2, 2, 2, 3, 10. 14. 2, 2, 2, 2, 2, 13. 15. 2, 3, 3, 5, 11. 16. 2, 2, 2, 3, 3, 7. 17. 3, 5, 29. 18. 2, 3, 37. 19. 2, 2, 2, 2, 3, 7. 20. 5, 5, 31. 21. 17, 17. 22. 2, 2, 2, 2, 2, 5, 5. 23. 3, 5, 5, 5, 7. 24. 2, 2, 3, 3, 89. 25. 3, 5, 7, 11. 26. 2, 2, 5, 5, 5, 5. 27. 3, 3, 3, 5, 11. 28. 2, 2, 2, 2, 3, 3, 3. 29. 2, 2, 2, 2, 2, 3, 3, 7. 30. 5, 5, 5, 23. 31. 2, 5, 17, 17.

Page 142.—3. 2 $\frac{1}{2}$. 4. 1 $\frac{1}{2}$.

Page 143.—5. 4. 6. 16. 7. 3. 8. 27 $\frac{1}{2}$. 9. 40. 10. 18 $\frac{1}{2}$. 11. 45. 12. 6. 18. 4. 14. 60. 15. 40. 16. 3 $\frac{1}{2}$. 17. 16 $\frac{1}{2}$. 18. 6 $\frac{1}{2}$. 19. 8. 20. 2 $\frac{1}{2}$. 21. 1 $\frac{1}{2}$. 22. 3 $\frac{1}{2}$. 23. 6 $\frac{1}{2}$. 24. $\frac{1}{2}$. 25. 1. 26. 15 $\frac{1}{2}$ yd. 27. \$860. 28. \$3.12 $\frac{1}{2}$. 29. \$3.12.

Page 144.—30. 48 $\frac{1}{2}$. 31. 1 $\frac{1}{2}$. 32. \$48. 33. 53 $\frac{1}{2}$ yd. 34. 7. 35. 39 $\frac{1}{2}$. 36. 2 $\frac{1}{2}$. 37. 161 $\frac{7}{15}$. 38. 1 $\frac{1}{2}$. 39. 4 $\frac{1}{2}$. 40. 13 $\frac{1}{2}$.

Page 151.—2. 1 $\frac{1}{2}$. 3. 7 $\frac{1}{2}$. 4. 5 $\frac{1}{2}$. 5. 1 $\frac{1}{2}$. 6. 2 $\frac{1}{2}$. 7. 2 $\frac{1}{2}$. 8. 1 $\frac{1}{2}$. 9. 4 $\frac{1}{2}$. 10. 2 $\frac{1}{2}$. 11. 5 $\frac{1}{2}$. 12. 4 $\frac{1}{2}$. 13. 2 $\frac{1}{2}$. 14. 3 $\frac{1}{2}$. 15. 2 $\frac{1}{2}$. 16. 7 $\frac{1}{2}$. 17. 1 $\frac{1}{2}$. 18. 1 $\frac{1}{2}$. 19. 2 $\frac{1}{2}$. 20. 2 $\frac{1}{2}$. 21. 5 $\frac{1}{2}$.

Page 152.—2. 17 $\frac{1}{2}$. 3. 42 $\frac{1}{2}$. 4. 49 $\frac{1}{2}$. 5. 46 $\frac{1}{2}$. 6. 52 $\frac{1}{2}$. 7. 22 $\frac{1}{2}$. 8. 16 $\frac{1}{2}$. 9. 12 $\frac{1}{2}$. 10. 22 $\frac{1}{2}$. 11. 20 $\frac{1}{2}$. 12. 14 $\frac{1}{2}$. 13. 19 $\frac{1}{2}$. 14. 21 $\frac{1}{2}$. 15. 3 $\frac{1}{2}$. 16. 10 $\frac{1}{2}$. 17. 15 $\frac{1}{2}$. 18. 24 $\frac{1}{2}$. 19. 49 $\frac{1}{2}$. 20. 56 $\frac{1}{2}$. 21. 93 $\frac{1}{2}$.

Page 154.—2. 1. 3. $\frac{1}{2}$. 4. $\frac{1}{2}$. 5. $\frac{1}{2}$. 6. $\frac{1}{2}$. 7. $\frac{1}{2}$. 8. $\frac{1}{2}$. 9. $\frac{1}{2}$. 10. $\frac{1}{2}$. 11. $\frac{1}{2}$. 12. $\frac{1}{2}$. 13. $\frac{1}{2}$. 14. $\frac{1}{2}$. 15. $\frac{1}{2}$. 16. $\frac{1}{2}$. 17. $\frac{1}{2}$. 18. $\frac{1}{2}$. 19. $\frac{1}{2}$. 20. $\frac{1}{2}$. 21. $\frac{1}{2}$. 22. $\frac{1}{2}$. 23. $\frac{1}{2}$. 24. $\frac{1}{2}$. 25. $\frac{1}{2}$.

Page 155.—2. $\frac{1}{2}$. $\frac{1}{2}$. $\frac{1}{2}$. 3. $\frac{1}{2}$. $\frac{1}{2}$. $\frac{1}{2}$. 4. $\frac{1}{2}$. $\frac{1}{2}$. $\frac{1}{2}$. 5. $\frac{1}{2}$. $\frac{1}{2}$. $\frac{1}{2}$. 6. $\frac{1}{2}$. $\frac{1}{2}$. $\frac{1}{2}$. 7. $\frac{1}{2}$. $\frac{1}{2}$. $\frac{1}{2}$. 8. $\frac{1}{2}$. $\frac{1}{2}$. $\frac{1}{2}$. 9. $\frac{1}{2}$. $\frac{1}{2}$. $\frac{1}{2}$. 10. $\frac{1}{2}$. $\frac{1}{2}$. $\frac{1}{2}$. 11. $\frac{1}{2}$. $\frac{1}{2}$. $\frac{1}{2}$. 12. $\frac{1}{2}$. $\frac{1}{2}$. $\frac{1}{2}$. 13. $\frac{1}{2}$. $\frac{1}{2}$. $\frac{1}{2}$. 14. $\frac{1}{2}$. $\frac{1}{2}$. $\frac{1}{2}$.

Page 156.—15. $\frac{1}{2}$. $\frac{1}{2}$. $\frac{1}{2}$. 16. $\frac{1}{2}$. $\frac{1}{2}$. $\frac{1}{2}$. 17. $\frac{1}{2}$. $\frac{1}{2}$. $\frac{1}{2}$. 18. $\frac{1}{2}$. $\frac{1}{2}$. $\frac{1}{2}$. 19. $\frac{1}{2}$. $\frac{1}{2}$. $\frac{1}{2}$. 20. $\frac{1}{2}$. $\frac{1}{2}$. $\frac{1}{2}$. 21. $\frac{1}{2}$. $\frac{1}{2}$. $\frac{1}{2}$. 22. $\frac{1}{2}$. $\frac{1}{2}$. $\frac{1}{2}$. 23. $\frac{1}{2}$. $\frac{1}{2}$. $\frac{1}{2}$. 24. $\frac{1}{2}$. $\frac{1}{2}$. $\frac{1}{2}$. 25. $\frac{1}{2}$. $\frac{1}{2}$. $\frac{1}{2}$. 26. $\frac{1}{2}$. $\frac{1}{2}$. $\frac{1}{2}$. 27. $\frac{1}{2}$. $\frac{1}{2}$. $\frac{1}{2}$. 28. $\frac{1}{2}$. $\frac{1}{2}$. $\frac{1}{2}$.

Page 158.—2. 1 $\frac{1}{2}$. 3. 1 $\frac{1}{2}$. 4. 1 $\frac{1}{2}$. 5. 1 $\frac{1}{2}$. 6. 1 $\frac{1}{2}$. 7. 2 $\frac{1}{2}$. 8. 1 $\frac{1}{2}$. 9. 1 $\frac{1}{2}$. 10. 13 $\frac{1}{2}$. 11. 10 $\frac{1}{2}$. 12. 16 $\frac{1}{2}$. 13. 16 $\frac{1}{2}$. 14. 12 $\frac{1}{2}$. 15. 23 $\frac{1}{2}$. 16. 21 $\frac{1}{2}$. 17. 17 $\frac{1}{2}$. 18. \$85 $\frac{1}{2}$. 19. 28 $\frac{1}{2}$ bu. 20. 698 $\frac{1}{2}$ lb. 21. 125 $\frac{1}{2}$ ft. 22. 8 $\frac{1}{2}$ o'clock.

- Page 160.** — 3. $\frac{1}{4}$. 4. $\frac{1}{3}$. 5. $\frac{1}{5}$. 6. $\frac{1}{3}$. 7. $\frac{1}{2}$. 8. $\frac{1}{3}$. 9. $\frac{1}{2}$.
Page 161. — 10. $\frac{1}{3}$. 11. $\frac{1}{2}$. 12. $\frac{1}{3}$. 13. $\frac{1}{4}$. 14. $\frac{1}{2}$.
 15. $4\frac{1}{3}$. 16. $18\frac{1}{4}$. 17. $4\frac{1}{3}$. 18. $5\frac{1}{3}$. 19. $5\frac{1}{3}$. 20. $5\frac{1}{3}$. 21. $3\frac{1}{3}$.
 22. $10\frac{1}{2}$. 23. $23\frac{1}{2}$. 24. $71\frac{1}{2}$ lb. 25. $83\frac{1}{2}$ acres. 26. $6\frac{1}{2}$ ft.
 27. $30\frac{1}{2}$ lb. 28. $\$30\frac{1}{4}$. 29. $\frac{1}{2}$. 30. $\frac{1}{2}$. 31. $34\frac{1}{2}$ yd. 32. $\frac{1}{4}$.
Page 162. — 33. $1\frac{1}{2}$. 34. $2\frac{1}{2}$. 35. $\frac{1}{2}$. 36. $1\frac{1}{2}$. 37. $\frac{1}{2}$. 38. $\frac{1}{2}$.
 39. $\frac{1}{2}$. 40. $9\frac{1}{2}$. 41. $4\frac{1}{3}$. 42. $4\frac{1}{3}$. 43. $7\frac{1}{2}$. 44. $5\frac{1}{2}$.
 45. $4\frac{1}{3}$. 46. $10\frac{1}{2}$.
Page 163. — 2. $1\frac{1}{2}$. 3. $\frac{1}{2}$. 4. $\frac{1}{2}$. 5. $3\frac{1}{2}$. 6. $8\frac{1}{2}$. 7. 10.
 8. $10\frac{1}{2}$. 9. $12\frac{1}{2}$. 10. $10\frac{1}{2}$. 11. $12\frac{1}{2}$. 12. $20\frac{1}{2}$. 13. $78\frac{1}{2}$. 14. $158\frac{1}{2}$.
 15. $259\frac{1}{2}$. 16. $130\frac{1}{2}$.
Page 164. — 17. $71\frac{1}{2}$. 18. $247\frac{1}{2}$ mi. 19. $\$13.33\frac{1}{2}$. 20. $\$100\frac{1}{2}$.
 21. 165 ft. 22. $16\frac{1}{2}$ cords. 23. 175 hr. 24. $\$67\frac{1}{2}$.
Page 165. — 2. $6\frac{1}{2}$. 3. 15. 4. $24\frac{1}{2}$. 5. 21. 6. $12\frac{1}{2}$. 7. $20\frac{1}{2}$.
 8. $47\frac{1}{2}$. 9. $19\frac{1}{2}$. 10. $17\frac{1}{2}$. 11. $17\frac{1}{2}$. 12. $38\frac{1}{2}$. 13. $19\frac{1}{2}$. 14. $22\frac{1}{2}$.
 15. 85. 16. $225\frac{1}{2}$. 17. $241\frac{1}{2}$. 18. $21\frac{1}{2}$. 19. 528.
Page 166. — 20. $45\frac{1}{2}$. 21. 100 acres. 22. $\$249\frac{1}{2}$. 23. $\$5.61$.
 24. $452\frac{1}{2}$ mi. 25. $\$5.03\frac{1}{2}$. 26. $\$148\frac{1}{2}$.
Page 168. — 2. $\frac{1}{2}$. 3. $\frac{1}{2}$. 4. $\frac{1}{2}$. 5. $\frac{1}{2}$. 6. $\frac{1}{2}$. 7. $\frac{1}{2}$. 8. $\frac{1}{2}$.
 9. $\frac{1}{2}$. 10. $\frac{1}{2}$. 11. $\frac{1}{2}$. 12. $\frac{1}{2}$. 13. $\frac{1}{2}$. 14. 3. 15. $6\frac{1}{2}$.
 16. $5\frac{1}{2}$. 17. $6\frac{1}{2}$. 18. $\frac{1}{2}$. 19. $1\frac{1}{2}$. 20. $4\frac{1}{2}$. 21. $26\frac{1}{2}$. 22. $\frac{1}{2}$.
 23. $\frac{1}{2}$. 24. $\frac{1}{2}$. 25. $\frac{1}{2}$. 26. $14\frac{1}{2}$ ft.; $57\frac{1}{2}$ ft. 27. $\$12\frac{1}{2}$.
 28. $\$58\frac{1}{2}$. 29. $168\frac{1}{2}$ acres. 30. $\$145\frac{1}{2}$. 31. $\$1.81\frac{1}{2}$. 32. Lining
 $\$2\frac{1}{2}$; Dress $\$22\frac{1}{2}$.
Page 171. — 3. $\frac{1}{3}$. 4. $\frac{1}{4}$. 5. $\frac{1}{3}$. 6. $\frac{1}{2}$. 7. $\frac{1}{2}$. 8. $\frac{1}{3}$.
 9. $\frac{1}{4}$. 10. $\frac{1}{3}$. 11. $\frac{1}{2}$. 12. $\frac{1}{3}$. 13. $\frac{1}{2}$. 14. $\frac{1}{3}$. 15. $2\frac{1}{2}$.
 17. $3\frac{1}{2}$. 18. $3\frac{1}{2}$. 19. $3\frac{1}{2}$. 20. $5\frac{1}{2}$. 21. $5\frac{1}{2}$. 22. $17\frac{1}{2}$. 23. $19\frac{1}{2}$.
 24. $22\frac{1}{2}$. 25. $\$8\frac{1}{2}$. 26. $4\frac{1}{2}$ da. 27. $\$10$ per lb. 28. $\$791\frac{1}{2}$.
 29. $\$1\frac{1}{2}$. 30. $\frac{1}{2}$ da.
Page 172. — 31. $23\frac{1}{2}$ mi. per hr. 32. $12\frac{1}{2}$ sq. rd.
Page 173. — 2. 30. 3. $36\frac{1}{2}$. 4. $42\frac{1}{2}$. 5. $43\frac{1}{2}$. 6. 54. 7. $62\frac{1}{2}$.
 8. $42\frac{1}{2}$. 9. $29\frac{1}{2}$. 10. $79\frac{1}{2}$. 11. $62\frac{1}{2}$. 12. $72\frac{1}{2}$. 13. $76\frac{1}{2}$. 14. $47\frac{1}{2}$.
 15. $36\frac{1}{2}$. 16. $45\frac{1}{2}$. 17. $129\frac{1}{2}$. 18. $139\frac{1}{2}$. 19. $257\frac{1}{2}$. 20. $898\frac{1}{2}$.
 21. $1486\frac{1}{2}$. 22. $6\frac{1}{2}$. 23. 9. 24. $4\frac{1}{2}$. 25. $17\frac{1}{2}$. 26. $4\frac{1}{2}$.
 27. $7\frac{1}{2}$. 28. $4\frac{1}{2}$. 30. $12\frac{1}{2}$. 31. $7\frac{1}{2}$. 32. $12\frac{1}{2}$. 33. $9\frac{1}{2}$.
 34. $20\frac{1}{2}$. 35. $5\frac{1}{2}$. 36. $2\frac{1}{2}$. 37. $5\frac{1}{2}$. 38. $31\frac{1}{2}$ yd. 39. $23\frac{1}{2}$ rd.
 40. $21\frac{1}{2}$ bbl. 41. $36\frac{1}{2}$ bbl. 42. 3 cases.
Page 175. — 3. $1\frac{1}{2}$. 4. $1\frac{1}{2}$. 5. $2\frac{1}{2}$. 6. $3\frac{1}{2}$. 7. $1\frac{1}{2}$.
 8. $1\frac{1}{2}$. 9. $10\frac{1}{2}$. 10. $2\frac{1}{2}$. 11. $3\frac{1}{2}$. 12. $1\frac{1}{2}$. 13. $6\frac{1}{2}$.
 14. $1\frac{1}{2}$. 15. $5\frac{1}{2}$. 16. $5\frac{1}{2}$. 17. $40\frac{1}{2}$. 18. $30\frac{1}{2}$. 19. $4\frac{1}{2}$.
 20. $5\frac{1}{2}$.
Page 176. — 21. $12\frac{1}{2}$ times. 22. $7\frac{1}{2}$ bu. 23. $7\frac{1}{2}$ bbl.
 24. $4\frac{1}{2}$ baskets. 25. 8 bbl. 26. $7\frac{1}{2}$ lb. 27. 10 books. 28. $7\frac{1}{2}$ doz.
 29. 6 plows. 30. $51\frac{1}{2}$ acres. 31. $\$4056\frac{1}{2}$. 32. $5\frac{1}{2}$ bu. 33. $21\frac{1}{2}$ bu.

- Page 177.—35. $\frac{1}{15}$. 36. $\frac{1}{15}$. 37. $\frac{1}{15}$. 38. $\frac{1}{15}$. 39. $\frac{1}{15}$.
 40. $\frac{1}{15}$. 41. $\frac{1}{15}$. 42. $\frac{1}{15}$. 43. $\frac{1}{15}$. 44. $\frac{1}{15}$. 45. $\frac{1}{15}$. 46. $\frac{1}{15}$.
 2. $\frac{2}{15}$. 3. $\frac{1}{15}$. 4. $\frac{1}{15}$. 5. $\frac{1}{15}$. 6. $\frac{6}{15}$. 7. $\frac{11}{15}$. 8. $\frac{2}{15}$.
 9. $\frac{1}{15}$. 10. $\frac{1}{15}$. 11. $\frac{1}{15}$. 12. $\frac{1}{15}$. 13. $\frac{1}{15}$.
- Page 179.—2. $\frac{1}{15}$. 3. $\frac{1}{15}$. 4. $\frac{1}{15}$. 5. $\frac{1}{15}$. 6. $\frac{1}{15}$. 7. $\frac{1}{15}$.
 8. $\frac{1}{15}$. 9. $\frac{1}{15}$. 10. $\frac{1}{15}$. 11. $\frac{1}{15}$. 12. $\frac{1}{15}$. 13. $\frac{1}{15}$. 14. $\frac{1}{15}$.
 15. $\frac{1}{15}$. 16. $\frac{1}{15}$. 17. $\frac{1}{15}$. 18. $\frac{1}{15}$. 19. $\frac{1}{15}$. 20. $\frac{1}{15}$. 21. $\frac{1}{15}$. 22. $\frac{1}{15}$.
 Page 181.—2. 64. 3. 100. 4. 170. 5. 100. 6. 91. 7. 90.
 8. 325. 9. 400. 10. 600. 11. 850. 12. 380. 13. 510. 14. 5.
 15. $\frac{1}{15}$. 16. $\frac{1}{15}$. 17. $\frac{1}{15}$. 18. $\frac{1}{15}$. 19. $\frac{1}{15}$.
 Page 182.—20. \$120. 21. 100 ft. 22. 425 mi. 23. 150 ft.
 24. 192 plants. 25. 55 mi.
 Page 184.—1. $4\frac{1}{10}$. 2. $6\frac{1}{10}$. 3. $16\frac{1}{10}$. 4. $78\frac{1}{10}$. 5. $4\frac{1}{10}$.
 6. $42\frac{1}{10}$. 7. 765. 8. $34\frac{1}{10}$.
 Page 185.—9. $\$84\frac{1}{2}$. 10. 40 bu. 11. $\$25\frac{1}{2}$. 12. $211\frac{1}{2}$ bu.
 13. $\$4083\frac{1}{2}$. 14. $\$6$. 15. $\frac{1}{2}$. 16. 72 yr. 17. A, $\frac{1}{2}$ of work;
 B, $\frac{1}{2}$ of work; both, $\frac{1}{2}$ of work. 18. 42 da.; $\frac{1}{2}$ of 42 da. = $3\frac{1}{2}$ da.
 Page 186.—19. \$150,000. 20. $1\frac{1}{2}$ da. 21. $4\frac{1}{2}$ da. 22. 24 da.
 23. \$620; \$465. 24. $2\frac{1}{2}$ da. 25. \$90,000. 26. $14\frac{1}{2}$. 27. $21\frac{1}{2}$.
 28. 1. 29. $8\frac{1}{2}$. 30. \$312.50. 31. \$9000. 32. \$26,666.
 33. 719 girls; 320 boys.
 Page 187.—34. \$25,771 $\frac{1}{2}$. 35. $1\frac{1}{2}\frac{1}{2}$. 36. $20\frac{1}{4}\frac{1}{2}$ mi.
 37. $3\frac{1}{2}$ in.; $19\frac{1}{2}$ in. 38. \$610. 39. \$250,000. 40. $1\frac{1}{4}\frac{1}{2}$.
 41. Increased $\frac{1}{2}$. 42. 600 lb. 43. $\frac{1}{2}$ per lb. 44. \$34,600.
 45. $37\frac{1}{2}$ da.
 Page 192.—2. $\frac{1}{2}$. 3. $\frac{1}{2}$. 4. $\frac{1}{2}$. 5. $\frac{1}{2}$. 6. $\frac{1}{2}$. 7. $\frac{1}{2}$. 8. $\frac{1}{2}$.
 9. $\frac{1}{2}$. 10. $\frac{1}{2}$. 11. $\frac{1}{2}$. 12. $\frac{1}{2}$. 13. $\frac{1}{2}$. 14. $\frac{1}{2}$. 15. $\frac{1}{2}$.
 16. $\frac{1}{2}$. 17. $\frac{1}{2}$.
 Page 193.—2. .3. 3. .625. 4. .1875. 5. .25. 6. .106+.
 7. .15. 8. .025. 9. .0606+. 10. .316+. 11. .7105263+.
 12. .45+. 13. .108+. 14. .428571+. 15. .36585+. 16. .0064.
 17. 3.5. 18. 1.867142+. 19. 8.66+. 20. 3.166+. 21. 6.35.
 22. .57+. 23. .16025. 24. .0375. 25. .006.
 2. 84.331. 3. 68.454. 4. 8.301. 5. 53.875.
 Page 194.—6. 38.6635. 7. 64.435. 8. 5.1195. 9. 91.7472.
 10. 430.435. 11. 8.615. 12. 49.1964. 13. 35.89262. 14. 67.8399.
 15. 80.81. 16. 1.2119. 17. .9062498. 18. 1.938. 19. .3706883.
 20. 133 miles. 21. \$38,741.875.
 Page 195.—2. 11.33. 3. 10.095. 4. 27.995. 5. 25.46. 6. 12.012.
 7. 36.213. 8. 25.2963. 9. 5.4798. 10. 4.08. 11. 8.81. 12. 5.2905.
 13. 4.1125. 14. .585. 15. .161739. 16. 26.6299. 17. 12.81965.
 18. 76.5466. 19. 6.3066. 20. 4.75 bbl. 21. \$25.75. 22. \$2624.13.
 Page 196.—2. .1875. 3. .816. 4. 38.76. 5. 1.545. 6. .04871.
 7. .34125. 8. .3285. 9. 1.19075. 10. 3.971.

- Page 197.** — 11. .018394. 12. 4.02732. 13. 99.8959. 14. 1.3509.
 15. 17.92431. 16. .0180655. 17. .00163644. 18. .001252394.
 19. .0082022. 20. .595875. 21. 53.9852. 22. .051255. 24. 276.5.
 25. 496.75. 26. 279.5. 27. 893.5. 28. 687.35. 29. 376.8. 30. 683.5.
 31. 763.4. 32. 899.2; 8992; 89920; 899200. 33. 6830.25; 68302.5;
 683025. 34. .765625 Sum greater. 35. 23.7375. 36. 3.03125.
 37. 513.60. 38. \$30.50. 39. 391.865 acres. 40. 400.2 mi.
- Page 198.** — 2. 17.2811. 3. 11759.6. 4. 98.765. 5. \$26.06.
 6. \$129.62. 7. \$40,598. 8. \$5.875. 10. \$7.35. 11. \$7.455.
 12. 98.0625. 13. 58.545. 14. 138.668. 15. 1.4405. 16. 9.54.
 17. .769. 18. \$655.60. 19. \$4408.375.
- Page 200.** — 2. 21.7. 3. 3.9487+. 4. 3420. 5. 3.84. 6. 8710.
 7. 67.2. 8. .47. 9. 3.4. 10. .036. 11. 51.2. 12. 376. 13. .0036.
 14. .0131. 15. 1.461. 16. .167104+. 17. .213. 18. 5.5. 19. .036.
 20. 2500. 21. .0072. 22. .315. 23. 2,000,000. 24. 51.6. 25. 2.34.
 26. 100.2. 27. 64.2. 28. 6275. 29. .0496+. 30. 32,820.
 31. 1,000,000,000.
- Page 201.** — 33. 3.925. 34. 2.645. 35. .3695. 36. .04825.
 37. .03862. 38. .004285. 39. 26.7 yd. 40. 14.75 bbl. 41. 265.8 cords.
- Page 202.** — 2. \$21.82. 3. \$1564.50. 4. \$10.20. 5. \$31.405.
- Page 203.** — 1. \$30. 2. \$785. 3. \$58.6625. 4. \$84.50.
 5. 7000 ft. 6. \$61.394. 7. \$4.875. 8. 8.368+ yr. 9. 1820 letters.
 10. 36 pairs. 11. \$54.25. 12. \$121.25.
- Page 204.** — 13. 13.5 hr. 14. \$17.07. 15. \$105.25. 16. 20%.
 17. \$338. 18. \$1544.70. 19. \$0.2 $\frac{1}{2}$ gain, \$59.00. 20. \$2360.
 21. 1155.95, tons sold; 159.05, tons melted. 22. 12 hr.
- Page 215.** — 2. 176 in. 3. 451 in. 4. 574 in. 5. 1007 in.
 6. 220 gi. 7. 436 gi. 8. 494 gi. 9. 607 gi. 10. 15,315 sec.
 11. 36,738 sec. 12. 66,035 sec. 13. 6728 oz. 14. 14,012 oz.
 15. 73,292 oz. 16. 139,769 oz. 17. 26,362 gr. 18. 48,248 gr.
 19. 31,569 gr. 20. 251 pt. 21. 367 pt. 22. 409 pt. 23. 930 in.
 24. 1082 in. 25. 1654 in. 26. 4482 sq. in. 27. 7732 sq. in.
 28. 18,012 $\frac{1}{2}$ sq. yd. 29. 39,480 $\frac{1}{2}$ sq. yd. 30. 110,677 cu. in.
 31. 197,706 cu. in. 32. 18,680 gr. 33. 27,220 gr. 34. 1698 sheets.
 35. 2122 sheets. 36. 20,400 gr. 37. 33,660 gr. 38. 555,900 sec.
 39. 575 pt. 40. 1758 in. 41. 1980 pt. 42. 28,438 sq. yd.
 43. 463,482 in. 45. 37 lb. 8 oz. 46. 3 yd. 1 ft. 3 $\frac{1}{4}$ in. 47. 5 da.
 10 hr. 40 min. 48. 1 ft. 3 $\frac{1}{4}$ in. 49. 2 pk. 3.2 qt. 50. 9 cwt.
 51. 10 cu. ft. 1382.4 cu. in. 52. 266 rd. 3 yd. 2 ft. 53. 68 sq. rd.
 17 sq. yd. 2 sq. ft. 82 $\frac{1}{2}$ sq. in. 54. 5 da. 2 hr. 38 min. 24 sec.
 55. 1 qt. 2 gi. 56. 1 ft. 5.1 in.
- Page 216.** — 3. 131 gal. 2 qt. 1 pt. 1 gi. 4. 398 gal. 1 pt. 5. 178
 gal. 3 qt. 1 pt. 6. 17 rd. 4 $\frac{1}{2}$ yd. 11 in. 7. 12 rd. 2 yd. 8 in.
 8. 8 rd. 1 ft. 9 in.
- Page 217.** — 9. 55 bu. 2 pk. 6 qt. 10. 73 bu. 5 qt. 1 pt. 11. 61 bu.
 2 pk. 6 qt. 1 pt. 12. 3 cwt. 22 lb. 12 oz. 13. 5 cwt. 7 lb. 15 oz.
 14. 2 T. 19 cwt. 83 lb. 15. 1 T. 17 cwt. 95 lb. 16. 25 wk. 3 da.
 13 hr. 17. 2 hr. 16 min. 8 sec. 18. 6 da. 20 hr. 54 min. 19. 3 sq. yd.
 55 sq. in. 20. 4 sq. yd. 5 sq. ft. 33 sq. in. 21. 3 R. 18 qr. 22.
 22. 4 R. 16 qr. 11 sheets. 23. 4 cu. ft. 952 cu. in. 24.

1 pwt. 10 gr. 25. 1 lb. 5 oz. 9 pwt. 10 gr. 26. 1 lb. 7 oz. 18 gr.
28. $\frac{1}{2}$ yd. 29. $\frac{1}{3}$ pk. 30. 1 $\frac{1}{4}$ bu. 31. $\frac{1}{4}$ gal. 32. $\frac{1}{4}$ bu. 33. $\frac{1}{4}$ rd.

Page 218. — 3. 35 gal. 1 qt. 8. 38 bu. 2 pk. 4. 22 da. 13 hr.
30 min. 5. 169 lb. 14 oz. 6. 35 lb. 8 oz. 17 pwt. 21 gr. 7. 70 yd.
2 ft. 9 in. 8. 84 gal. 1 qt. 1 pt. 9. 79 A. 158 sq. rd. 10. 41 T. 31 lb.
11. 23 yd. 1 ft. 10 $\frac{1}{4}$ in. 2. 5 bu. 1 pk. 4 qt.

Page 219. — 3. 6 gal. 2 qt. 1 pt. 3 gi. 4. 2 da. 22 hr. 10 min.
5. 22 lb. 10 oz. 8 pwt. 6. 10 yd. 2 ft. 9 in. 7. 3 rd. 4 $\frac{1}{2}$ yd. 2 ft.
8. 5 lb. 9 oz. 6 dr. 9. 10 A. 68 sq. rd. 10. 25 sq. yd. 1 sq. ft. 95 sq. in.
11. 14 T. 17 cwt. 39 lb. 11 oz. 13. 51 yr. 5 mo. 5 da. 14. 25 yr.
6 mo. 18 da. 15. 52 yr. 2 mo. 29 da. 16. 10 yr. 10 mo. 17 da.

Page 220. — 2. 34 bu. 1 pk. 3 qt. 3. 34 gal. 1 pt. 4. 44 lb. 4 oz.
2 pwt. 16 gr. 5. 64 lb. 1 oz. 2 sc. 10 gr. 6. 16 hr. 42 min. 55 sec.
7. 18 T. 3 cwt. 91 lb. 8 oz. 8. 24 rd. 2 yd. 1 ft. 6 in. 9. 75 cu. yd.
12 cu. ft. 1506 cu. in. 10. 41 sq. yd. 2 sq. ft. 81 sq. in. 11. 21 da.
9 hr. 1 min. 40 sec. 12. 103 rd. 1 ft. 10 in. 13. 48 R. 18 qr.
14. 44 bbl. 18 gal. 1 pt.

3. 3 gal. 2 qt. 1 pt. 1 $\frac{1}{2}$ gi. 3. 3 bu. 3 pk. 7 qt. $\frac{1}{2}$ pt. 4. 1 yd. 2 ft.
11 $\frac{1}{2}$ in. 5. 4 cwt. 78 lb. 11 $\frac{1}{2}$ oz. 6. 4 lb. 8 oz. 19 pwt. 10 $\frac{1}{2}$ gr.
7. 2 lb. 8 oz. 7 dr. 2 sc. 2 $\frac{1}{2}$ gr. 8. 3 hr. 11 min. 46 sec. 9. 5 sq. yd.
4 sq. ft. 11 $\frac{1}{2}$ sq. in. 10. 3 rd. 2 ft. 11 $\frac{1}{2}$ in.

Page 222. — 3. 391 sq. ft. 3. 192 sq. ft. 21 $\frac{1}{2}$ sq. yd. 4. 48 sq. yd.
5. 19,200 sq. rd.; 120 A. 6. 30 A. 7. \$17,578+. 8. \$27.14.
9. \$25.50. 10. 240 A. 11. \$60. 13. 15 A. corn; 7 $\frac{1}{2}$ A. potatoe.
Page 224. — 2. 420 cu. ft. 3. 792 cu. in. 4. 400 cu. yd.
5. 2100 cu. ft. 6. 680 cu. ft. 7. 888 cu. ft. 8. 1260 cu. ft.
9. 984 $\frac{1}{2}$ cu. ft. 10. 700 cu. ft. 11. 8 cd. 12. 8 od. 13. 72 cd.
14. \$396.

Page 225. — 15. \$78,222+. 16. \$36,863+.
1. 24 ft. 2. 15 ft. 3. 18 $\frac{1}{2}$ ft. 4. 23 $\frac{1}{2}$ ft. 5. 12 $\frac{1}{2}$ ft. 6. 26 $\frac{1}{2}$ ft.
7. \$3,00. 8. 1440 ft. 9. \$25.92. 10. \$18,426+. 11. \$25.38.

Page 227. — 2. \$25,480+. 3. \$85,224 4. \$97,275. 5. \$61,376.
6. \$64,832. 7. \$174,60.

Page 228. — 9. \$43,968. 10. \$66,855+. 11. \$93,907+.
12. \$171,971+. 13. \$180,742+. 14. \$40,853+. 15. \$125,147+.
17. \$47,281+; Amt. \$407,521+. 18. \$85,865+; Amt. \$501,345+.
19. \$22,098+; Amt. \$239,458+. 20. \$94,195+; Amt. \$517,545+.
21. \$130,729+; Amt. \$850,889+. 22. \$70,774+; Amt. \$539,134+.
23. \$32,262+; Amt. \$251,732+. 24. \$25,494+; Amt. \$350,834+.
25. \$91,508+; Amt. \$527,838+. 26. \$94,44+; Amt. \$391,89+.

Page 229. — 1. \$250. 2. Answers differ. 3. 4 yd. 1 ft. 10 in.
4. 9 bu. 2 pk. 5 qt. 5. 1885 $\frac{1}{2}$ revolutions. 6. 95 $\frac{1}{2}$. 7. 10 children.
8. $\frac{1}{4}$. 9. 583 $\frac{1}{2}$ bu. 10. 4 $\frac{1}{2}$ da. 11. \$28250. 12. 40 $\frac{1}{2}$; 25 $\frac{1}{2}$.

Page 230. — 13. \$36. 14. 105 A. 145 sq. rd. 15. 75 $\frac{1}{2}$ A.
16. \$7.87 amount of bill. 17. 5 da. 5 h. 38 min. 20 sec. 18. 453 $\frac{1}{2}$ mi.
19. 2160 ft. 20. \$37,453+. 21. \$161,906+. 22. 5 lb. 2 oz. 23. 48
suits. 24. \$40. 25. 42 yd.; \$29,40. 26. 64 rd. long.

Page 231. — 27. \$582,40. 28. 210; 280. 29. \$62500. 30. \$633.
31. \$418. 32. \$55,20. 33. 2880 pills. 34. \$29. 35. \$5,04.
7. \$4125. 60. 37. \$39,20. 38. $\frac{1}{4}$. 39. 7 $\frac{1}{4}$ mi. 40. 134.

1 lb. 2 oz.

